SUBRIEMANNIAN GEOMETRY AND BEYOND
19^{th}–23^{rd} FEBRUARY 2018
JYVÄSKYLÄ, FINLAND
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TITLES AND ABSTRACTS

On a computer, you can click on the title and be redirected to the abstract.

Monday	
9:00–10:00	Andrei Agrachev (SISSA, Trieste): *Homotopically visible and invisible singular curves*
10:00–10:30	Coffee Break
10:30–11:30	Ludovic Rifford (Cóte d'Azur, Nice): *Sard conjectures and measures contractions properties in sub-Riemannian geometry*
11:30–12:30	Luca Rizzi (Institut Fourier, Grenoble): *Sub-Riemannian interpolation inequalities*
12:30–14:00	Lunch
14:00–15:30	Activities on Ice
15:30–16:00	Coffee Break
16:00–17:00	Luis Guijarro (Autonoma, Madrid): *Symmetries in metric spaces*
17:00–18:00	Daniele Morbidelli (Bologna): *New examples of John and uniform domains in Carnot–Carathéodory spaces*
19:30– ∞	Social dinner in Hotel Alba

Tuesday	
9:00–10:00	Alessandro Ottazzi (UNSW, Sydney): *Contactomorphisms between polarised groups*
10:00–10:30	Coffee Break
10:30–11:30	Anton Lukyanenko (GMU): *Number theory in rational Carnot groups*
11:30–12:00	Scott Zimmerman (UCONN): *Applications of a change of variables for Lipschitz mappings into metric spaces (Short Talk)*
12:00–12:30	Conference Photo
12:30–14:00	Lunch
14:00–15:00	Tony Liimatainen (Helsinki): *On the Popp extension of a horizontal metric on subRiemannian manifolds and definitions of quasiconformality*
15:00–15:30	Andrei Ardentov (RAS): *Cut locus in Euler's elastic problem (Short Talk)*
15:30–16:30	Coffee Break
16:30–17:30	Andrea Malchiodi (SNS, Pisa): *On the Sobolev quotient in CR geometry*

Wednesday	
9:00–10:00	Fässler Katrin (Fribourg): *Intrinsic Lipschitz graphs and quantitative rectifiability in the Heisenberg group*
10:00–10:30	Coffee Break
10:30–11:30	Sean Li (UCONN): *Traveling salesman and singular integrals*
11:30–12:30	Laura Venieri (Helsinki): *A characterization of extremal sets for the dimension comparison in the Heisenberg group*
12:30–14:00	Lunch
At 14:00	Bus to Tupaswilla
Afternoon	Excursion to Tupaswilla
Evening	Social dinner in Tupaswilla

Thursday	
9:00–10:00	Pierre Pansu (Paris Sud, Orsay): *$\ell^{q,p}$ cohomology of certain Carnot groups*
10:00–10:30	Coffee Break
10:30–11:30	Andrea Pinamonti (Trento): *Maximal directional derivatives and universal differentiability sets in Carnot groups*
11:30–12:00	Derek Jung (Illinois): *BiLipschitz embeddings into jet space Carnot groups (Short Talk)*
12:00–12:30	Sebastiano Don (Padova): *Some properties of BV functions on Carnot-Carathéodory spaces (Short Talk)*
12:30–14:00	Lunch
14:00–15:00	Sylvester Eriksson-Bique (UCLA): *Non-self similar carpets in higher dimensions and general metric spaces*
15:00–15:30	Eero Hakavuori (Jyväskylä): *Tangent and asymptotic cones of geodesics in Carnot groups (Short Talk)*
15:30–16:30	Coffee Break
16:30–17:30	Jesus Jaramillo (Complutense, Madrid): *Existence and uniqueness of ∞-harmonic functions under assumption of ∞-Poincaré inequality*

Friday	
9:00–10:00	Jeremy Tyson (UIUC): *Heat content in the Heisenberg group*
10:00–10:30	Coffee Break
10:30–11:30	Michael Cowling (UNSW, Sydney): *Mappings of groups that preserve reflections*
11:30–12:00	Ludovic Sacchelli (CMAP): *Whitney C^1 extension of horizontal curves in sub-Riemannian manifolds (Short Talk)*
12:00–12:30	
12:30–14:00	Lunch
14:00–15:00	Alessio Martini (Birmingham): *Sub-elliptic harmonic analysis*
15:00–16:00	Benjamin Warhurst (Warsaw): *A Koebe distortion theorem for quasiconformal mappings in the Heisenberg group*
16:00–17:00	Coffee End

Monday, 19 February 2018

Homotopically visible and invisible singular curves

Andrei Agrachev
(SISSA, Trieste)

Let M be a Riemannian manifold equipped with a bracket generating vector distribution $\Delta \subset TM$ and let $\Omega_\Delta \subset H^1([0,1]; M)$ be the space of integral curves of the distribution. The *boundary map*:

$$\partial_\Delta : \Omega_\Delta \to M \times M, \qquad \partial_\Delta(\gamma) = (\gamma(0), \gamma(1)),$$

is smooth; *singular curves* are just critical points of this map.

We study the influence of singular curves on the homotopy type of Lebesgue sets of the action functional on the generalized loop spaces $\partial_\Delta^{-1}(q_0, q_1)$ in order to develop a Morse theory for these spaces. The role of singular curves is very different for rank 2 distributions and for distributions of rank greater than 2. This is a joint work with Francesco Boarotto and Antonio Lerario.

Sard conjectures and measures contractions properties in sub-Riemannian geometry

Ludovic Rifford
(Côte d'Azur, Nice)

After recalling a few facts on the Sard Conjecture and the minimizing Sard Conjecture in Sub-Riemannian geometry, I will address the validity of Measure Contraction Properties in Sub-Riemannian manifolds. This property consists in quantifying the way the volumes are contracted along minimizing geodesics toward a point, it is related with the regularity properties of the horizontal gradients of geodesic distances.

Sub-Riemannian interpolation inequalities

Luca Rizzi
(Institut Fourier, Grenoble)

Sub-Riemannian structures can be described as limits of Riemannian ones with $\mathrm{Ric}(g_n) \to -\infty$ and they represent, in a certain sense, the most singular case among the three great classes of geometries (Riemannian, Finlser, and sub-Riemannian ones). In this talk, we discuss how, under generic assumptions, these structures support interpolation inequalities à la Cordero-Erasquin-McCann-Schmuckenschläger. As a byproduct, we characterize the sub-Riemannian cut locus as the set of points where the squared sub-Riemannian distance fails to be semiconvex. Specifying our results to the case of the Heisenberg groups, we recover in an intrinsic way the inequalities recently obtained by Balogh, Kristály and Sipos. As a further application, we obtain new and sharp results on the measure contraction properties of the standard Grushin structure. The techniques are based on optimal transport and sub-Riemannian Jacobi fields. Joint work with Davide Barilari.

Symmetries in metric spaces

Luis Guijarro
(Autonoma, Madrid)

Understanding a metric space is easier if it has a good amount of symmetries; compare for instance what is known about homogeneous spaces to what can be said of arbitrary Riemannian manifolds. In this talk, we will examine the symmetry groups of some metric spaces that have acquired big popularity during the last twenty years: Alexandrov spaces (or metric spaces with lower sectional curvature bounds), and a far reaching refinement of Lott-Otto-Villani spaces: the so called $RCD(K, N)$-spaces, introduced by Ambrosio-Gigli and Savare as a replacement of manifold with lower curvature bounds.

New examples of John and uniform domains in Carnot–Carathéodory spaces

Daniele Morbidelli
(Bologna)

We consider a class of vector fields in $\mathbb{R}^3$ (of arbitrarily large step) and we discuss conditions ensuring that a given open set is a John or uniform domain with respect to the Carnot–Carathéodory distance. Furthermore, we discuss the Ahlfors-regularity of the perimeter measure on the boundary. Joint work with Roberto Monti and Daniele Gerosa.

Tuesday, 20 February 2018

Contactomorphisms between polarised groups

Alessandro Ottazzi
(UNSW, Sydney)

Let S be a Lie group and consider a left-invariant subbundle $H(S)$ of its tangent bundle that is bracket generating. We call the pair $(S, H(S))$ a polarised group. A Carnot group $(G, H(G))$ is a stratified group with polarisation defined by the first layer of the stratification. A diffeomorphism f between open subsets of S and G is a contactomorphism if it maps $H(S)$ onto $H(G)$. Given a Carnot group $(G, H(G))$, we construct groups $(S, H(S))$ for which a local contactomorphism exists. If G ha finite dimensional Tanaka prolongation, our construction is exhaustive. This work is in collaboration with Sebastiano Nicolussi Golo.

Number theory in rational Carnot groups

Anton Lukyanenko
(GMU)

In many ways, the structure of Carnot groups parallels that of Euclidean space, and in particular of the real line, making it reasonable to generalize number-theoretic ideas to the Carnot group setting.

Rational Carnot groups admit integer subgroups and dilations, and thus one can study fractions, base-b expansions, and Diophantine approximation on these spaces. Furthermore, Heisenberg-type Carnot groups admit a well-behaved Koranyi inversion, which allows one to define continued fraction expansions, which lead to connections with hyperbolic geometry and a surprisingly distinct notion of Diophantine approximation.

I will discuss previous work on these topics with Vandehey, as well as new ergodicity results that were initially motivated by the Heisenberg group, but are new even in the case of complex continued fractions.

Applications of a change of variables for Lipschitz mappings into metric spaces
(Short Talk)

Scott Zimmerman
(UCONN)

Consider a C^1 mapping $f : \mathbb{R}^k \to \mathbb{R}^n$, and suppose $Df(x_0)$ has rank j for $j \leq k \leq n$. It follows from the classical implicit function theorem that there is a C^1 diffeomorphism Φ in a neighborhood of x_0 so that the mapping $f \circ \Phi^{-1}$ fixes the first j coordinates in a neighborhood of $\Phi(x_0)$. This result extends easily to mappings $f : \mathbb{R}^k \to \ell^\infty$ whose coordinates are C^1 functions and hence, by way of the Kuratowski embedding and C^1-Lusin approximations, to any metric space valued Lipschitz mapping on $\mathbb{R}^k$. This was first introduced in a paper by Malekzadeh and Hajłasz. In this talk, I will present applications of this change of variables to BLD

mappings, Sard-type results in metric spaces, and sub-Riemannian geometry. This covers joint work with Soheil Malekzadeh and with Piotr Hajłasz.

On the Popp extension of a horizontal metric on subRiemannian manifolds and definitions of quasiconformality

Tony Liimatainen
(Helsinki)

We study the Popp extension of a horizontal metric on equiregular subRiemannian manifolds. The Popp extension is a locally defined Riemannian metric, whose volume form is the Popp volume form. We show that the Popp extension can be used to give a differential type of definition of quasiconformality similar to that used on Riemannian manifolds. One benefit of this definition is that it reduces the calculation of quasiconformality constant to straightforward eigenvalue computations related to the horizontal differential of the mapping. The Popp extension was recently studied also by D. Barilari and L. Rizzi.

Cut locus in Euler's elastic problem (Short Talk)

Andrei Ardentov
(RAS)

The classical problem on stationary configurations of an elastic planar rod is considered. Length of the rod, ends of the rod and directions at the ends are fixed. In 1694, Jacob Bernoulli connected the problem with a general equation for curvature. Later in 1742, his nephew Daniel made the first clear mathematical statement of the problem as a variational problem in terms of potential energy and proposed the problem to Leonard Euler who classified all possible configurations of the rod - Euler's elasticae. It is known that sufficiently small pieces of Euler's elasticae are optimal, i.e. have minimum of potential energy. In theory, the point, where an optimal curve loses its optimality, is called a cut point. The set of all cut points is called a cut locus. The talk presents new results and hypotheses about structure of the cut locus in Euler's elastic problem.

On the Sobolev quotient in CR geometry

Andrea Malchiodi
(SNS, Pisa)

We consider a class of three-dimensional CR manifolds which are modelled on the Heisenberg group. We prove positivity of the mass under the condition that the Webster curvature is positive and that the manifold is embeddable. We apply this result to the CR Yamabe problem, and we discuss the properties of Sobolev-type quotients, giving some counterexamples for Rossi spheres. This is joint work with J.H.Cheng and P.Yang.

Wednesday, 21 February 2018

Intrinsic Lipschitz graphs and quantitative rectifiability in the Heisenberg group

Katrin Fässler
(Fribourg)

I will discuss steps towards the extension of the theory of uniform rectifiability, developed by G. David and S. Semmes, from Euclidean spaces to the sub-Riemannian Heisenberg group (for sets of codimension 1). A special focus will lie on the role played by intrinsic Lipschitz graphs. The talk is based on joint work with Vasileios Chousionis, Tuomas Orponen, and Séverine Rigot.

Traveling salesman and singular integrals

Sean Li
(UCONN)

In 1977, Calderon proved that the Cauchy transform is bounded as a singular integral operator on the L_2 space of Lipschitz graphs in the complex plane. This subsequently sparked much work on singular integral operators on subsets of Euclidean space. Due to the deep works of David, Jones, Semmes, and many others, it is now known that the boundedness of singular integrals of certain odd kernels is intricately linked to a rectifiability structure of the underlying sets.

The 1-rectifiability of sets in the Heisenberg group has an almost tight characterization via an analyst's traveling salesman theorem which measures deviations of the set from "horizontal lines". We use this to study the connection between singular integrals and rectifiability for 1-dimensional subsets of the Heisenberg group where we find a similar connection to the Euclidean case. However, the kernels studied turn out to be positive and even, in stark contrast with the Euclidean setting. We also describe a recent 1-sided extension of the traveling salesman theorem to general Carnot groups.

Based on multiple joint works with V. Chousionis, R. Schul, and S. Zimmerman.

A characterization of extremal sets for the dimension comparison in the Heisenberg group

Laura Venieri
(Helsinki)

Given a set in the first Heisenberg group, it is known that its Hausdorff dimension with respect to a left-invariant and one-homogeneous metric can vary in a certain range depending on its Euclidean Hausdorff dimension. We prove that sets whose dimension is the minimal or maximal possible satisfy certain geometric properties: in the first case they are in a sense horizontal and in the second case vertical. We also show the sharpness of our results with some examples. This talk is based on a joint work with P. Mattila.

Thursday, 22 February 2018

$\ell^{q,p}$ cohomology of certain Carnot groups.

Pierre Pansu
(Paris Sud, Orsay)

Every closed L^p differential form on Euclidean n-space has a primitive in L^q provided $1/p-1/q=1/n$. We prove analogous results for Carnot groups, which allows to compute $\ell^{q,p}$-cohomology, a large scale invariant of topological flavour. Joint work with Annalisa Baldi, Bruno Franchi and Michel Rumin.

Maximal directional derivatives and universal differentiability sets in Carnot groups

Andrea Pinamonti
(Trento)

Rademacher's theorem asserts that Lipschitz functions from $\mathbb{R}^n$ to $\mathbb{R}^m$ are differentiable almost everywhere. Such a theorem may not be sharp: if $n > 1$ then there exists a Lebesgue null set N in $\mathbb{R}^n$ containing a point of differentiability for every Lipschitz mapping $f : \mathbb{R}^n \to \mathbb{R}$. Such sets are called universal differentiability sets and their construction relies on the fact that existence of an (almost) maximal directional derivative implies differentiability. We will see that maximality of directional derivatives implies differentiability in all Carnot groups where the Carnot–Caratheodory distance is suitably differentiable, which include all step 2 Carnot groups (in particular the Heisenberg group). Further, one may construct a

measure zero universal differentiability set in any step 2 Carnot group. Finally, we will observe that in the Engel group, a Carnot group of step 3, things can go badly wrong. Based on joint work with Enrico Le Donne and Gareth Speight.

BiLipschitz embeddings into jet space Carnot groups
(Short Talk)

Derek Jung
(Illinois)

For all $k, n \geq 1$, we construct a biLipschitz embedding of $\mathbb{S}^n$ into the jet space Carnot group $J^k(\mathbb{R}^n)$ that does not admit a Lipschitz extension to $\mathbb{B}^{n+1}$, extending work of Rigot-Wenger from 2010. The hope is that (as in the Heisenberg case) such an embedding could be used to prove the non-density of Lipschitz mappings in certain jet space-valued Sobolev spaces. We first choose a smooth, positive function $f : \mathbb{B}^n \to \mathbb{R}$ with k^{th}-order derivatives approximately linear near $\partial\mathbb{B}^n$. Realizing $\mathbb{S}^n$ as two copies of $\mathbb{B}^n$, the embedding is given by taking the jet of f on the upper hemisphere and the jet of $-f$ on the lower hemisphere. Finally, we modify an argument of Hajłasz-Schikorra-Tyson for $n = 1$ and generalize an argument of Rigot-Wenger for $n \geq 2$ to prove the lack of a Lipschitz extension.

Some properties of BV functions on Carnot-Carathéodory spaces
(Short Talk)

Sebastiano Don
(Padova)

In this talk we are going to analyze some properties of the distributional derivative of a BV function in a Carnot-Carathéodory space. In particular we are going to show that in any equiregular CC space supporting De Giorgi rectifiability theorem a BV function is approximately differentiable almost everywhere, its jump set is rectifiable and its measure derivative satisfies a decomposition formula that is analogous to the Euclidean case.
Finally we are going to present the Rank-One theorem for BV functions in a class of Carnot groups including all Heisenberg groups $\mathbb{H}^n$ with $n \geq 2$.
The results are obtained in collaboration with Annalisa Massaccesi and Davide Vittone.

Non-self similar carpets in higher dimensions and general metric spaces

Sylvester Eriksson-Bique
(UCLA)

I will discuss ongoing work with Jasun Gong on giving a general framework to prove results similar to Tyson, McKay and Wildrick on the existence and explicit examples of non-selfsimilar subsets of Euclidean space supporting Poincare inequalities. Our work gives a single proof that works for a variety of carpet- and spongelike metric spaces in the plane and higher dimensions, and shows that many positive measure subsets with empty interior support $(1, p)$-Poincare inequalities for p>1. We suggest that such constructions could also give examples of subsets of the Heisenberg Group with empty interior satisfying Poincare inequalities. Further they lead to interesting classification problems in the plane. The results lead to new examples but also give a new and rather short way of proving the results.

Tangent and asymptotic cones of geodesics in Carnot groups (Short Talk)

Eero Hakavuori
Jyväskylä

The tangent cone of a curve contains all the possible tangents of the curve at some fixed point. The differentiability problem of geodesics in sub-Riemannian geometry can thus be considered via the study of tangent cones. The dilation structure of Carnot groups is particularly convenient for this study, since the tangents of geodesics become themselves (infinite) geodesics in the same Carnot group. For this reason also the asymptotic properties of infinite geodesics are of interest, since such properties will give restrictions on the possible tangents of geodesics. In this talk I will discuss our recent results that every tangent of a geodesic is also a geodesic in a Carnot group of lower step and that every asymptotic cone of a geodesic is also a geodesic in a Carnot subgroup of lower rank. This is joint work with Enrico Le Donne.

Existence and uniqueness of ∞-harmonic functions under assumption of ∞-Poincaré inequality

Jesus Jaramillo
(Complutense, Madrid)

We introduce a natural notion of ∞-harmonic functions defined on a bounded domain of a metric measure space. If X is a complete metric measure space X whose measure is doubling and supports an ∞-Poincaré inequality, and Ω is a bounded domain in X, for every Lipschitz function $f : \partial\Omega \to \mathbb{R}$ we obtain the existence and uniqueness of an ∞-harmonic extension of f to Ω. To do so, we show that in our setting there is a metric that is bi-Lipschitz equivalent to the original one, and such that with respect to this new metric the space satisfies the so-called ∞-weak Fubini property. Furthermore, we also show that if the metric satisfies the ∞-weak Fubini property, then the notion of ∞-harmonic functions coincide with the notion of AMLEs proposed by Aronsson. We also show that the notion of ∞-harmonicity is in general distinct from the notion of strongly absolutely minimizing Lipschitz extensions considered by Crandall-Evans-Gariepy or Juutinen, but they coincide when the metric space supports a p-Poincaré inequality for some finite $p \geq 1$. This is a joint work with Estibalitz Durand-Cartagena and Nages Shamungalingam.

Friday, 23 February 2018

Heat content in the Heisenberg group

Jeremy Tyson
(UIUC)

I will discuss short-time asymptotics for the total heat content of a smoothly bounded domain with noncharacteristic boundary in the first Heisenberg group H^1. Our asymptotic expansion of the total heat content involves standard sub-Riemannian notions of perimeter and horizontal mean curvature. The proofs rely on an explicit description of tubular Carnot-Carathéodory neighborhoods of surfaces in H^1 (Arcozzi-Ferrari, 2007-2008; Ritoré, 2017). Our results generalize prior work in the Riemannian setting by van den Berg, Gilkey and Le Gall. This talk is based on joint work with Jing Wang.

Mappings of groups that preserve reflections

Michael Cowling
(UNSW, Sydney)

Suppose that G is a group, considered as a homogeneous space of itself. The natural reflection $x \mapsto x^{-1}$ around the identity may be transported by translations to a reflection ρ_y around any point y. Then a bijection $\Phi : G \to H$ of groups preserves reflections if and only if $\Phi(yx^{-1}y) = \Phi(y)\Phi(x)^{-1}\Phi(y)$ for all $x, y \in G$. We describe all such mappings in the case where G and H are connected locally compact groups.

Whitney C^1 extension of horizontal curves in sub-Riemannian manifolds

(Short Talk)

Ludovic Sacchelli
(CMAP)

We discuss the Whitney C^1 extension property for horizontal curves in sub-Riemannian manifolds that satisfy a first-order Taylor expansion compatibility condition. We show that the extension property holds true whenever a suitable non-singularity property holds for the endpoint map on the Carnot groups obtained by nilpotent approximation. This happens in particular for step-2 manifolds. As an application, we obtain a Lusin-like approximation theorem for horizontal curves.
Joint work with Mario Sigalotti.

Sub-elliptic harmonic analysis

Alessio Martini
(Birmingham)

Let L be the Laplacian on $\mathbb{R}^n$, or a more general elliptic operator on a manifold. The investigation of necessary and sufficient conditions for an operator of the form $F(L)$ to be bounded on L^p in terms of smoothness properties of the spectral multiplier F is a classical and very active research area of harmonic analysis, with long-standing open problems, such as the Bochner–Riesz conjecture. A number of sharp results are nonetheless available, which fundamentally reflect the underlying geometry.

In the presence of a sub-Riemannian geometric structure, the natural substitute L for the Laplacian need not be an elliptic operator, and it may be just sub-elliptic. This has dramatic consequences, both geometric and analytic, and even the simplest questions related to the L^p-boundedness of operators of the form $F(L)$ are far from being completely understood.

I will survey recent results dealing with the case of sub-Laplacians on 2-step Carnot groups, complex and quaternionic spheres, and Grushin operators.

A Koebe distortion theorem for quasiconformal mappings in the Heisenberg group

Benjamin Warhurst
(Warsaw)

The talk will discuss recent work with Tomasz Adomowicz and Katrin Fässler where we prove a Koebe distortion theorem for the average derivative of a quasiconformal mapping between domains in the sub-Riemannian Heisenberg group $\mathbb{H}^1$.