SUBRIEMANNIAN GEOMETRY AND BEYOND. II

18^{th}–22^{nd} FEBRUARY 2019

JYVÄSKYLÄ, FINLAND

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TITLES AND ABSTRACTS

On a computer, you can click on the title and be redirected to the abstract.
On Monday morning, the talks are in Agora: Auditorium 1.
On Monday afternoon, the talks are in Old Campus: Historia 320.
Afterwards the talks are in Agora: Auditorium 2.

Monday	
9:00–10:00	Séverine Rigot (Nice): *Quantitative rectifiability in the Heisenberg groups*
10:00–10:30	Coffee Break
10:30–11:30	Tuomas Orponen (Helsinki): *News on uniform rectifiability in the Heisenberg group*
11:30–12:30	Francesco Serra Cassano (Trento): *The Bernstein problem for area-minimizing intrinsic graphs in the sub-Riemannian Heisenberg group*
12:30–13:30	Lunch
13:30–15:00	Activities on Ice
15:00–16:00	Walk to old campus & Coffee Break
16:00–17:00	Yuri Sachkov (Pereslavl): *Sub-Finsler problems on the Cartan and Engel groups*
17:00–18:00	Emmanuel Trélat (UPMC): *Spectral analysis of sub-Riemannian Laplacians and Weyl measure*
18:00–19:00	Cocktail party at old campus

Tuesday	
9:00–10:00	Davide Vittone (Padova): *Rectifiability issues in sub-Riemannian geometry*
10:00–10:30	Coffee Break
10:30–11:30	Richard Montgomery (UCSC): *Elastica, Optimal Rolling, and Wong's equations*
11:30–12:00	Sebastiano Nicolussi Golo (Padova): *Spectral multipliers and wave equation for sub-Laplacians*
12:00–12:30	Conference Photo
12:30–14:00	Lunch
14:00–15:00	David Bate (Helsinki): *A non-linear Besicovitch-Federer type projection theorem for metric spaces*
15:00–15:30	Gabriel Pallier (Orsay): *On sublinearly quasisymmetric homeomorphisms*
15:30–16:30	Coffee Break & Poster Session
16:30–17:30	Guy C. David (BSU): *Quantitative flatness for curves in metric spaces*

Wednesday	
9:00–10:00	Bruno Franchi (Bologna): *Poincaré and Sobolev inequalities for differential forms in Heisenberg groups*
10:00–10:30	Coffee Break
10:30–11:30	Roberto Monti (Padova): *Some new results on stable hypersurfaces in the complex projective space and complex hyperbolic space*
11:30–12:30	Manuel Ritoré (Granada): *Tubular neighborhoods in the sub-Riemannian Heisenberg groups*
12:30–14:00	Lunch
At 14:30	Bus to Tupaswilla
Afternoon	Excursion to Tupaswilla
Evening	Social dinner in Tupaswilla

Thursday	
9:00–10:00	Stefan Wenger (Fribourg): *Constructing Hölder maps to Carnot groups*
10:00–10:30	Coffee Break
10:30–11:30	Piotr Hajłasz (Pittsburgh): *Hölder continuous mappings, differential forms and the Heisenberg groups*
11:30–12:00	Alexey Mashtakov (Eindhoven): *Sub-Riemannian Geometry in Image Processing*
12:00–12:30	Ville Kivioja: *On the classification of Lie groups by metric geometry*
12:30–14:00	Lunch
14:00–15:00	Igor Zelenko (TAMU): *Projective and affine equivalence of sub-Riemannian metrics: toward integrability and separation of variables conjectures*
15:00–15:30	Karen Habermann (Bonn): *Small-time fluctuations for a model class of hypoelliptic diffusion bridges*
15:30–16:30	Coffee Break & Poster Session
16:30–17:30	Frédéric Jean (ENSTA): *On the regularity of abnormal minimizers for rank 2 sub-Riemannian structures*

Friday	
9:00–10:00	Davide Barilari (IMJ-PRG): *Interpolation inequalities and comparison in sub-Riemannian geometry*
10:00–10:30	Coffee Break
10:30–11:30	Brian Street (UW): *Convenient Coordinates*
11:30–12:30	Luca Capogna (WPI): *A Liouville type theorem for quasiconformal maps between contact sub-Riemannian manifolds and applications*
12:30–14:00	Lunch
16:00–17:00	Coffee End

Monday, 18 February 2019

Quantitative rectifiability in the Heisenberg groups

Séverine Rigot
Nice

Several different notions of quantitative rectifiability in the sub-Riemannian Heisenberg groups have been recently used as useful tools that fit in a suitable way the intrinsic geometry of such spaces. Such notions appear in particular in recent works by Naor and Young, and by Fässler, Orponen and myself, and each one of them mimics one of the aspects of the theory of uniform rectifiability developed by David and Semmes in Euclidean spaces. In this talk, I will show links between these various notions, in order to bring them together in a single, unified picture.

News on uniform rectifiability in the Heisenberg group

Tuomas Orponen
Helsinki

To what extent can one develop a David-Semmes type theory of uniform rectifiability in the Heisenberg group? For example: how to quantify the rectifiability of intrinsic Lipschitz graphs, and are singular integrals bounded on them? I report on the progress, focusing on what has happened since the previous GeoMeG conference, and describe the main current challenges. The talk is based on joint work with V. Chousionis, K. Fässler, and S. Rigot.

The Bernstein problem for area-minimizing intrinsic graphs in the sub-Riemannian Heisenberg group

Francesco Serra Cassano
Trento

We will deal with the so-called *Bernstein problem* for area-minimizing intrinsic graphs in the first Heisenberg group $\mathbb{H}^1 \equiv (\mathbb{R}^3, \cdot)$, understood as a Carnot group and equipped by the sub-Riemannian metric structure. More precisely, the problen reads as follows: if the intrinsic graph $\Gamma_f \subset \mathbb{H}^1$ of a function $f : \mathbb{R}^2 \to \mathbb{R}$, that is

$$\Gamma_f := \left\{(0, y, t) \cdot (f(y, t), 0, 0) : (y, t) \in \mathbb{R}^2\right\},$$

is (locally) area minimizing in $\mathbb{H}^1$, then must Γ_f be a plane, in the geometry of $\mathbb{H}^1$? We will positively and negatively answer to this problem, taking the regularity of f into account.

Sub-Finsler problems on the Cartan and Engel groups

Yuri Sachkov
Pereslavl

Left-invariant ℓ_∞ sub-Finsler problems on the Cartan and Engel groups are considered as time-optimal problems. Different types of geodesics are characterized: abnormal, singular, bang-bang, and mixed. Abnormal and singular geodesics are optimal. For bang-bang and mixed minimizers, an upper bound on the number of switchings is proved. As a consequence, we prove that any two points in the state space can be connected by a piecewise smooth minimizer with a uniformly bounded number of smooth pieces. This is a joint work with Andrei Ardentov and Enrico Le Donne.

Spectral analysis of sub-Riemannian Laplacians and Weyl measure

Emmanuel Trélat
UPMC

In a series of works on sub-Riemannian geometry with Yves Colin de Verdière and Luc Hillairet, we study spectral properties of sub-Riemannian Laplacians, which are hypoelliptic operators. The main objective is to obtain quantum ergodicity results, what we have achieved in the 3D contact case. In the general case we study the small-time asymptotics of sub-Riemannian heat kernels. We prove that they are given by the nilpotentized heat kernel. In the equiregular case, we infer the local and microlocal Weyl law, putting in light the Weyl measure in sR geometry. This measure coincides with the Popp measure in low dimension but differs from it in general. We prove that spectral concentration occurs on the shief generated by Lie brackets of length $r-1$, where r is the degree of nonholonomy. In the singular case, like Martinet or Grushin, the situation is more involved but we obtain small-time asymptotic expansions of the heat kernel and the Weyl law in some cases.

Tuesday, 19 February 2019

Rectifiability issues in sub-Riemannian geometry

Davide Vittone
Padova

In this talk we discuss two problems concerning "rectifiability" in sub-Riemannian geometry and particularly in the model setting of Carnot groups. The first problem regards the rectifiability of boundaries of sets with finite perimeter in Carnot groups, while the second one concerns Rademacher-type results (existence of a tangent plane out of a negligible set) for (intrinsic) graphs with (intrinsic) Lipschitz regularity. We will introduce both problems and discuss the state-of-the-art. Eventually, we will present some recent results about the rectifiability of sets with finite perimeter in a certain class of Carnot groups (including the simplest open case, i.e., the Engel group) and about a Rademacher theorem for intrinsic Lipschitz graphs of any dimension in Heisenberg groups.

Elastica, Optimal Rolling, and Wong's equations

Montgomery Richard
UC Santa Cruz

Elastica are a class of extremal curves explored by Euler and which make sense on any surface Σ of constant curvature. We prove that elastica are solutions to Wong's equations for a non-Abelian charged particle traveling on the surface under the influence of Yang-Mills field whose structure group G is the isometry group of an auxilary constant curvature surface Σ_2.

This result is closely linked to another characterization of elastica as the projections of optimal rolling curves for rolling Σ_2 along Σ. These optimal rolling curves are subRiemannian geodesics on the configuration space for rolling, a five-manifold which forms a circle bundle over $\Sigma \times \Sigma_2$.

The relation between elastica and optimal rolling curves was established by Jurdjevic-Simmons, then later by Grong, and is reproved here.

Spectral multipliers and wave equation for sub-Laplacians

Sebastiano Nicolussi Golo
Padova

The aim of this talk is to make the abstract clear to the whole audience.

Mihlin–Hörmander theorem gives the sharp Sobolev order $n/2$ for a spectral multiplier of the Laplacian to define a bounded operator on $L^p(\mathbb{R}^n)$ for all $p \in (1, \infty)$. We study the same type of statements for sub-Laplacians, which are sub-elliptic operators defined on sub-Riemannian manifolds.

Although a Mihlin–Hörmander-type theorem in Carnot groups is known, the sharp Sobolev order is still unknown. It is conjectured to be $n/2$, where n is the topological dimension.

We have proven that in no sub-Riemannian manifold the sharp Sobolev order can be lower than $n/2$, where n is the topological dimension. For the proof, we construct a partial representation of the sub-Riemannian half-wave propagator as a Fourier integral operator. For such Fourier integral operator, the critical points of the phase function are determined by the sub-Riemannian exponential map.

This is a joint work with Alessio Martini and Detlef Müller.

A non-linear Besicovitch-Federer type projection theorem for metric spaces

David Bate
Helsinki

We characterise purely n-unrectifiable subsets S of a complete metric space with finite n-dimensional Hausdorff measure by studying non-linear projections (i.e. 1-Lipschitz functions) into some fixed Euclidean space. We will show that a typical (in the sense of Baire category) non-linear projection maps S to a set of zero n-dimensional Hausdorff measure. Conversely, a typical non-linear projection maps an n-rectifiable subset to a set of positive n-dimensional Hausdorff measure.

These results provide a replacement for the classical Besicovitch-Federer projection theorem, which is known to be false outside of Euclidean spaces.

On sublinearly quasisymmetric homeomorphisms

Gabriel Pallier
Orsay

This short talk will be dedicated to the sublinearly quasisymmetric mappings introduced in my thesis. I will explain how they compare to the quasisymmetric mappings, where they come from, how to produce many of them on Euclidean domains, and finally address the question whether they exist (i.e., do they carry invariants?), between certain pairs of self-similar metric spaces.

Quantitative flatness for curves in metric spaces

Guy David
Ball State University

We'll survey some analogs of Peter Jones's "Analyst's Traveling Salesman Theorem" established by many authors in a variety of metric settings, both sub-Riemannian and beyond. Then we'll present new work, joint with Raanan Schul, that provides a sharp analog of the "upper bound" in this theorem in a general doubling metric space, showing that rectifiable curves in these settings are quantitatively locally flat in a precise way. We'll also explain a few corollaries of this result.

Wednesday, 20 February 2019

Poincaré and Sobolev inequalities for differential forms in Heisenberg groups

Bruno Franchi
Bologna

In this talk we prove contact Poincaré and Sobolev inequalities in Heisenberg groups $\mathbb{H}^n$, where the word "contact" is meant to stress that de Rham's exterior differential is replaced by the "exterior differential" d_c of the so-called Rumin's complex $(E_0^\bullet, d_c)$. A crucial feature of Rumin's construction is that d_c recovers the scale invariance of the "exterior differential" d_c under the group dilations associated with the stratification of the Lie algebra of $\mathbb{H}^n$. These inequalities provide a natural extension of the corresponding usual inequalities for functions in $\mathbb{H}^n$ and are a quantitative formulation of the fact that d_c-closed forms are locally d_c-exact. Joint work with A. Baldi and P. Pansu)

Some new results on stable hypersurfaces in the complex projective space and complex hyperbolic space

Roberto Monti
Padova

We present some new results on the problem of classifying compact hypersurfaces in the complex projective space and in the complex hyperbolic space that have constant mean curvature and are stable for deformations preserving the enclosed volume. In the case of the complex projective space, we prove that a stable hypersurface satisfying a certain bound on the curvatures must be a geodesic sphere. The results are joint work with Battaglia, Righini and Montefalcone.

Tubular neighborhoods in the sub-Riemannian Heisenberg groups

Manuel Ritoré
Granada

In this talk I will consider the Carnot-Carathéodory distance δ_E to a closed set E in a sub-Riemannian Heisenberg group $\mathbb{H}$, $n \geq 1$.The $\mathbb{H}$-regularity of δ_E is proved under mild conditions involving a general notion of singular point. In case E is a Euclidean submanifold of class C^k, $k \geq 2$, the distance function δ_E is C^k out of a singular set. In the case of hypersurfaces, a normal exponential map is explicitly described, allowing to obtain an explicit expression for the volume of the tubular neighborhood of E when the boundary ∂E is of class C^2. This expression is written in terms of the horizontal principal curvatures of ∂E and of the function $\langle N, T\rangle/|N_h|$ and its tangent derivatives.

Thursday, 21 February 2019

Constructing Hölder maps to Carnot groups

Stefan Wenger
Fribourg

Carnot groups equipped with a Carnot metric are subriemannian manifolds. These exhibit interesting (local) geometry that is far from Euclidean. In this talk we mainly focus on the special case of the first Heisenberg group H and study its geometry through Hölder mappings. By a theorem of Züst, which strengthens a result of Gromov and Pansu, every α-Hölder map with $\alpha > 2/3$ from a simply-connected Riemannian manifold to H factors through a metric tree. We show that Züst's result is sharp by constructing topologically non-trivial α-Hölder maps from the Euclidean 2-ball and 3-ball to H for every $\alpha < 2/3$. We use these to show that if $\alpha < 2/3$, then the set of α-Hölder maps from a compact metric space to H is dense in the set of continuous maps. Some of our results generalize to general Carnot groups. Joint work with Robert Young.

Hölder continuous mappings, differential forms and the Heisenberg groups

Piotr Hajłasz
Pittsburgh

Young defined Stielties integrals $\int_a^b f dg$ of Hölder continuous functions, $f, g \in C^{0,\alpha}$, $\alpha > 1/2$, and this theory plays an important role in stochastic calculus. More recently, three groups of researchers developed, completely independently, a multidimensional version of the Young integral $\int f dg_1 \wedge \ldots \wedge dg_n$ with different applications in mind: Brezis and Nguyen as an abstract theory; Conti, De Lellis, and Székelyhidi, in the context of convex integration; Züst in the setting of the Heisenberg groups. Since the functions are merely Hölder continuous, the theory can be regarded as the theory of distributional Jacobians of Hölder continuous maps.

I will talk about my recent joint work with Jacob Mirra and Armin Schikorra devoted to analysis of pullbacks of differential forms under Hölder continuous mappings with particular emphasis on applications to geometry and topology of the Heisenberg groups. While the approach of Züst was based on the theory of currents, our approach is more elementary and close to that of Brezis and Nguyen. We could find new proofs for many of the results of Züst, but also we could obtain new results. In particular we managed to find a simple proof of Gromov's non-embedding theorem and new results about Hölder homotopy groups of the Heisenberg groups.

Sub-Riemannian Geometry in Image Processing

Alexey Mashtakov
Eindhoven

In this talk we discuss how considering of sub-Riemannian (SR) structures on 2D and 3D images (or more precisely on their lift to the extended space of positions and directions) helps to detect some features, e.g. salient curves. We start from explanation of basic concepts of SR geometry and then show how they provide brain inspired methods in computer vision. We consider several particular examples: tracking of blood vessels in planar and spherical images of human retina, tracking of neural fibers in MRI images of human brain.

On the classification of Lie groups by metric geometry

Ville Kivioja
Jyväskylä

Riemannian and Subriemannian Lie groups are examples of what we call metric Lie groups: Lie groups that are endowed with left-invariant distances inducing the manifold topology. It is meaningful to ask, when two given Lie groups can be equipped with distances making them isometric metric spaces (as metric Lie groups). This geometric relation is possible only if the two Lie groups have the same large-scale geometry, i.e., are quasi-isometric. It is famously conjectured that two simply connected nilpotent Lie groups are quasi-isometric if and only if they are isomorphic. The same statement becames a theorem under the stronger geometric relation described above. In particular isometric Subriemannian nilpotent Lie groups are isomorphic. I will present a theorem describing the geometric relations on the class of solvable Lie groups of polynomial growth, a more general assumption than nilpotency. I will first recall the quasi-isometric classification of these groups in dimensions 3, 4 and 5, and then present the implications of our theorem in these dimensions.

Projective and affine equivalence of sub-Riemannian metrics: toward integrability and separation of variables conjectures

Igor Zelenko
TAMU

Two sub-Riemannian metrics are called projectively equivalent if they have the same geodesics up to a reparameterization and affinely equivalent if they have the same geodesics up to affine reparameterization. In the Riemannian case both equivalence problems are classical: local classifications of projectively and affinely equivalent Riemannian metrics were established by Levi-Civita in 1898 and Eisenhart in 1923, respectively. In particular, a Riemannian metric admitting a nontrivial (i.e. non-constant proportional) affinely equivalent metric must be a product of two Riemannian metrics i.e. certain separation of variable occur, while for the analogous property in the projectively equivalent case a more involved ("twisted") product structure is necessary. The latter is also related to the existence of sufficiently many commuting nontrivial integrals quadratic with respect to velocities for the corresponding geodesic flow. We will describe the recent progress toward the generalization of these classical results to sub-Riemannian metrics. In particular, we will discuss genericity of metrics that do not admit non-constantly proportional affinely/projectively equivalent metrics and the separation of variables on the level of linearization of geodesic flows (i.e. on the level of Jacobi curves) for metrics that admit non-constantly proportional affinely equivalent metrics. The talk is based on the collaboration with Frederic Jean (ENSTA, Paris) and Sofya Maslovskaya (INRIA, Sophya Antipolis).

Small-time fluctuations for a model class of hypoelliptic diffusion bridges

Karen Habermann
Bonn

We study the small-time asymptotics for hypoelliptic diffusion processes conditioned by their initial and final positions. After giving an overview of work on small-time asymptotics for sub-Riemannian diffusion bridges, we present recent results on small-time fluctuations for the bridge in a model class of diffusions satisfying a weak Hörmander condition, where the diffusivity is constant and the drift is linear. We show that, while the diffusion bridge can exhibit a blow-up behaviour in the small time limit, we can still make sense of suitably rescaled fluctuations which converge weakly.

On the regularity of abnormal minimizers for rank 2 sub-Riemannian structures

Frédéric Jean
ENSTA ParisTech

We prove the C^1 regularity for a class of abnormal length-minimizers in rank 2 sub-Riemannian structures. As a consequence of our result, all length-minimizers for rank 2 sub-Riemannian structures of step up to 4 are of class C^1. (Joint work with Davide Barilari, Yacine Chitour, Dario Prandi and Mario Sigalotti).

Friday, 22 February 2019

Interpolation inequalities and comparison in sub-Riemannian geometry

Barilari Davide
IMJ-PRG

In a celebrated paper of Cordero-Erasquin-McCann-Schmuckenschläger, the authors extends to the case of Riemannian manifolds some classical functional and geometrical interpolation inequalities in the Euclidean setting, using optimal transport techniques. In particular these results imply a "geodesic" version of the classical Brunn-Minkowski inequality in the Euclidean space. In this talk, we discuss the validity of such results in the case of sub-Riemannian geometry. In the second part we will discuss a new sub-Riemannian Bakry-Emery type concept of sectional and Ricci curvature, to take into account the ambient measure in comparison-type results. [Joint work with Luca Rizzi]

Convenient Coordinates

Brian Street
UW Madison

We discuss the method of picking a convenient coordinate system adapted to vector fields. Let $X_1, ..., X_q$be either real or complex C^1 vector fields. We discuss the question of when there is a coordinate system in which the vector fields are smoother (e.g., C^m, or C^∞, or real analytic). By answering this in a quantitative way, we obtain coordinate charts which can be used as generalized scaling maps. When the vector fields are real this is joint work with Stovall, and continues in the line of quantitative sub-Riemannian geometry initiated by Nagel, Stein, and Wainger. When the vector fields are complex one obtains a geometry with more structure which can be thought of as "sub-Hermitian".

A Liouville type theorem for quasiconformal maps between contact sub-Riemannian manifolds and applications

Luca Capogna
WPI

About 10 years, ago Michael Cowling conjectured that one could prove the smoothness up to the boundary of biholomorphisms between strictly pseudoconvex domains by using minimal regularity hypothesis, through an argument resting on ideas from the study of quasiconformal maps. In its simplest form, the proposed proof is articulated in two steps: (1) prove that any bi-holomorphism between smooth, strictly pseudoconvex domains extends to a homeomorphisms between the boundaries that is 1-quasiconformal with respect to the sub-Riemannian metric associated to the Levi form; (2) prove a Liouville type theorem, i.e. any $1-$quasiconformal homeomorphism between such boundaries is a smooth diffeomorphism.

In this talk I will discuss recent work with Le Donne, where we prove the first step of this program, as well as joint work with Citti, Le Donne and Ottazzi, where we settle the second step, thus concluding the proof of Cowling's conjecture. The proofs draw from several fields of mathematics, including nonlinear partial differential equations, and analysis in metric spaces.