Adapting to Non-Stationarity in EEG using a Mixture of Multiple ICA Models

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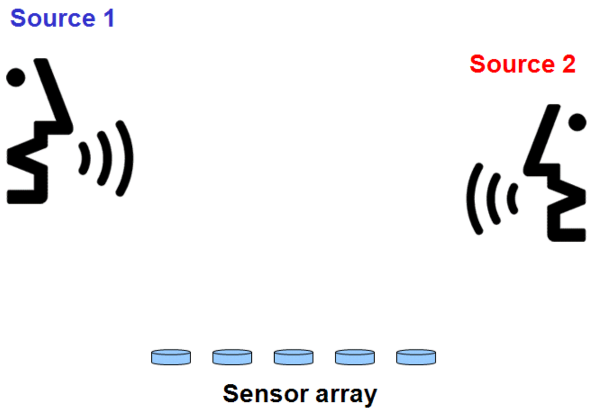
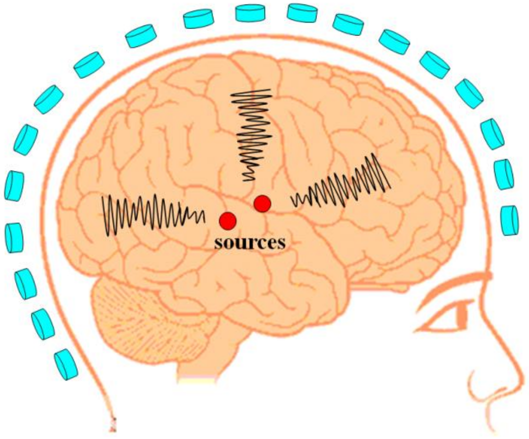
Introduction

- Want to model sensor array data with multiple independent sources — ICA





- Non-stationary source activity — mixture model
- Want the adaptation to be computationally efficient — Newton method

Outline

- Introduction
  - Non-stationarity in EEG
  - What is a mixture model?
- ICA Mixture Model
  - Model definition
  - Computational feasibility and Newton Method
- Examples
  - Application to epileptic seizure ECoG data
  - Application to typical EEG task data
- Implementation
  - Parallel computation

Non-stationarity

- What kinds of non-stationarity exist in EEG?
  - Environmental transients—lights, train, A/C
  - Different brain sources for different tasks
  - Muscle activity
  - Arousal level change
  - Seizure
- Are EEG components stable over recording? Which are and which are not?
- We approach this problem by using a mixture model of component bases with separate component maps and source statistics

What is a Mixture Model?

- A mixture model is a probabilistic combination of several models:

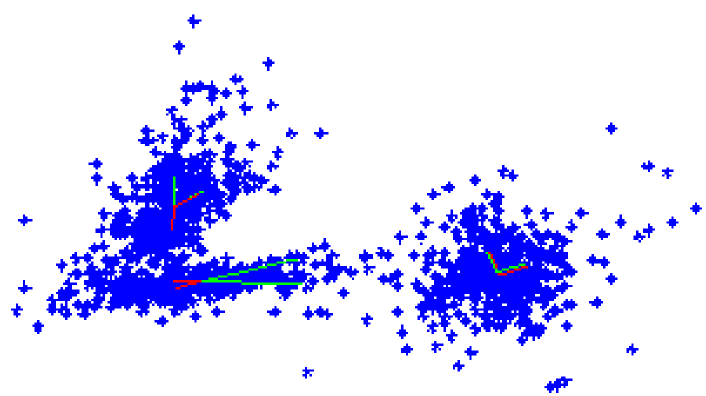
mixture proportions

means

$$p(x) = \sum_{j=1}^{M} \gamma_j \, p_j\left(\frac{x - \mu_j}{\sigma_j}\right)$$

scales

- Each data point modeled as being generated by one of the models in the mixture



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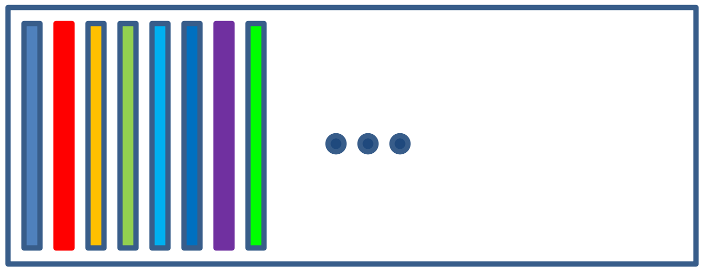
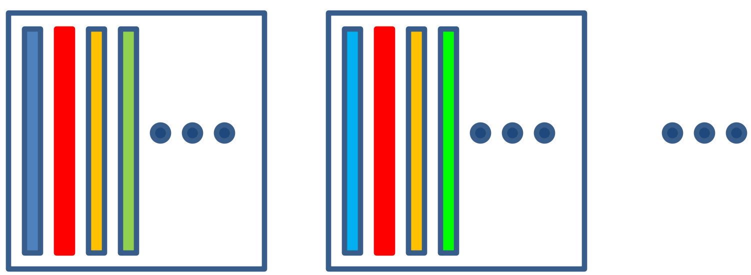
Mixture vs. Overcomplete

- Approach 1 – Overcomplete dictionary



- Approach 2 – Mixture of bases (like best basis selection)



- Assumptions:
  - At a given time at most num channels basis vectors present
  - Basis vectors do not combine arbitrarily but form subsets or groups of commonly occurring or mutually exclusive features

Computational Feasibility

- We will use an iterative algorithm, in which the basic steps are:
  - Estimate the independent source activations given models
  - Update models given estimated sources

- For large dimensional problems estimation of sources by iterative or even one-step methods takes non-trivial time, requiring inversion of a matrix for each sample
  - Example: data = 100 x 1,000,000, time to get sources = 1 ms per sample, one complete iteration takes at least 1000 seconds = 15 minutes, 500 iterations takes 6 days
  - Need iterations to be order seconds, so need source estimation to be very fast (less than 1ms) – use simple matrix multiplication, can't afford inversion

ICA Mixture Model

- Want to model observations $\boldsymbol{x}(t)$, $t = 1,\ldots,N$, different models "active" at different times
- Bayesian linear mixture model, $h = 1, \ldots, M$:

$$\mathbf{x}(t) = \mathbf{A}_h \mathbf{s}(t) + \mathbf{c}_h$$

- Conditionally linear given the model, $\mathbf{W}_h \triangleq \mathbf{A}_h^{-1}$:

$$p(\mathbf{x}(t) \,|\, h) = |\det \mathbf{W}_h| \, q_h\big(\mathbf{W}_h(\mathbf{x}(t) - \mathbf{c}_h)\big)$$

- Samples are modeled as independent in time:

$$p(\mathbf{X}; \Theta) = \prod_{t=1}^{N} \sum_{h=1}^{M} \gamma_h \, p(\mathbf{x}(t) \,|\, h)$$

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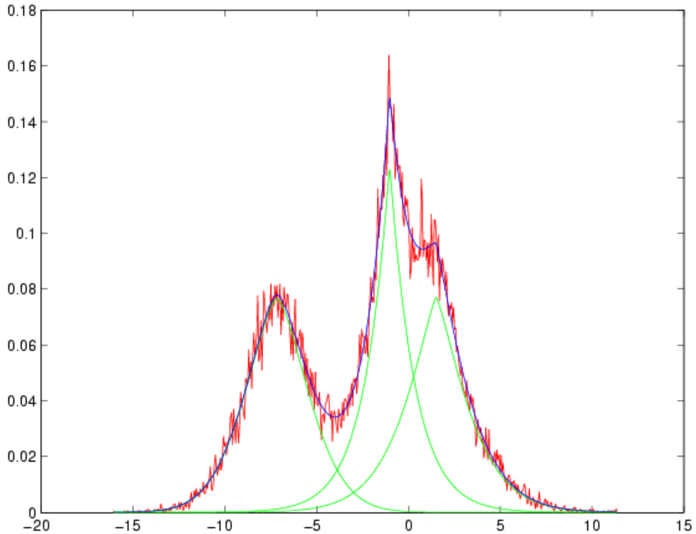
Source Density Mixture Model

- Each source density mixture component has unknown location, scale, and shape:

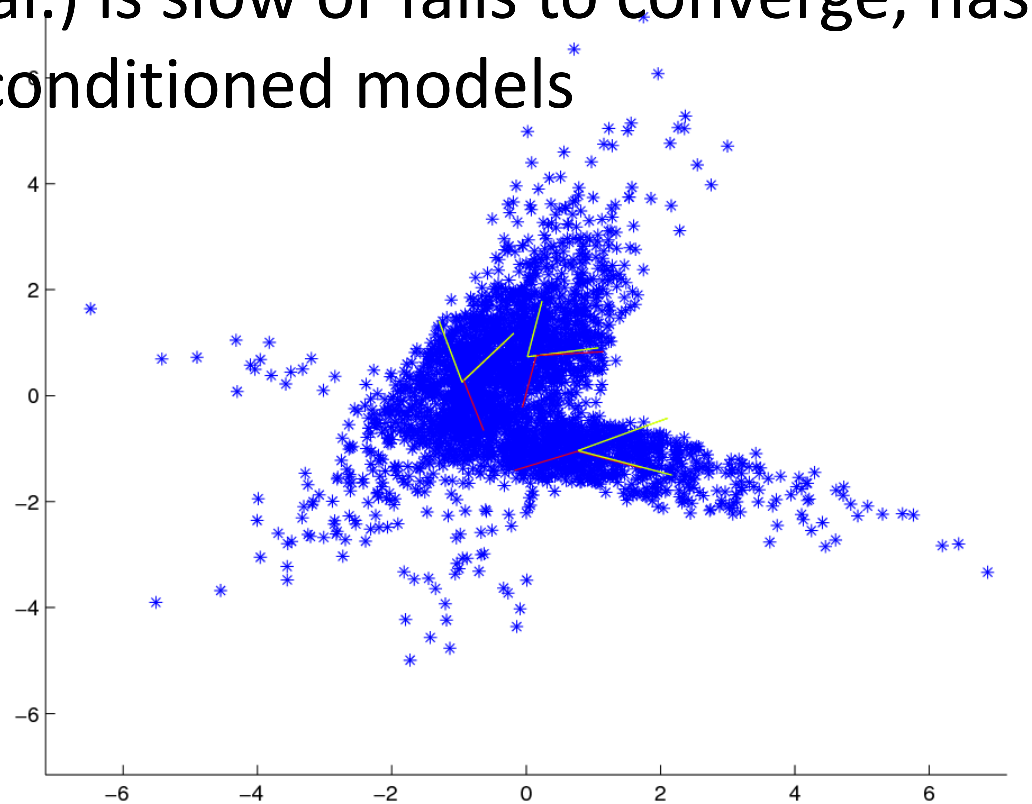
$$q_{hi}\big(s_i(t)\big) = \sum_{j=1}^{m} \alpha_{hij}\sqrt{\beta_{hij}}\, q_{hij}\Big(\sqrt{\beta_{hij}}\big(s_i(t)-\mu_{hij}\big)\,;\rho_{hij}\Big)$$

- Generalized Gaussian mixture model is convenient and flexible



Sub- and Super-Gaussian sources

- With mixture source model, model sources can be sub- or super-Gaussian, no need to check
- Newton method converges very fast, natural gradient (Lee et al.) is slow or fails to converge, has difficulty on poorly conditioned models



ICA Mixture Model—Invariances

- The complete set of parameters to be estimated is:

$$\Theta = \{\mathbf{W}_h, \mathbf{c}_h, \gamma_h, \alpha_{hij}, \mu_{hij}, \beta_{hij}, \rho_{hij}\}$$

$h = 1, \ldots, M, \quad i = 1, \ldots, n, \quad j = 1, \ldots, m$

- Invariances: **W** row norm/source density scale and model centers/source density locations:

$$[\mathbf{W}'_h]_{i:} = [\mathbf{W}_h]_{i:}/\tau_{hi},$$
$$\mu'_{hij} = \mu_{hij}/\tau_{hi}, \quad \beta'_{hij} = \beta_{hij}\tau_{hi}^2, \quad j = 1, \ldots, m$$

$$\mathbf{c}'_h = \mathbf{c}_h + \Delta\mathbf{c}_h, \quad \mu'_{hij} = \mu_{hij} - [\mathbf{W}_h \Delta\mathbf{c}_h]_i, \quad j = 1, \ldots, m$$

Basic ICA Newton Method

- Transform gradient (1st derivative) of cost function using inverse Hessian (2nd derivative)
- Cost function is data log likelihood:

$$p(\mathbf{X}) = \prod_{t=1}^{N} |\det \mathbf{W}|\, p_{\mathbf{s}}(\mathbf{W}\mathbf{x}_t)$$

$$L(\mathbf{W}) = \sum_{t=1}^{N} -\log|\det \mathbf{W}| + f(\mathbf{y}_t)$$

- Gradient:

$$\nabla L(\mathbf{W}) \propto -\mathbf{W}^{-T} + \frac{1}{N}\sum_{t=1}^{N} \nabla f(\mathbf{y}_t)\mathbf{x}_t^T$$

- Natural gradient (positive definite transform):

$$\Delta \mathbf{W} = \left(\mathbf{I} - \frac{1}{N}\sum_{t=1}^{N} \mathbf{g}_t \mathbf{y}_t^T\right)\mathbf{W}$$

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Newton Method – Hessian

- Take derivative of (*i,j*)th element of gradient with respect to (*k,l*)th element of $\mathbf{W}$:

$$\frac{\partial g_{ij}}{\partial w_{kl}} = [\mathbf{W}^{-1}]_{li}[\mathbf{W}^{-1}]_{jk} + \Big\langle f_i''\big([\mathbf{W}\mathbf{x}_t]_k\big)x_j x_l \delta_{ik}\Big\rangle_N$$

- This defines a linear transform $\mathbf{C} = \mathcal{H}(\mathbf{B})$:

$$c_{ij} = \sum_k \sum_l [\mathbf{W}^{-1}]_{li}[\mathbf{W}^{-1}]_{jk} b_{kl} + \Big\langle f_i''(y_i)\, x_j \sum_l b_{il} x_l \Big\rangle_N$$

- In matrix form, this is:

$$\mathbf{C} = \mathbf{W}^{-T}\mathbf{B}^T\mathbf{W}^{-T} + \frac{1}{N}\sum_{t=1}^{N} \mathrm{diag}(f''(\mathbf{y}_t))\mathbf{B}\mathbf{x}_t\mathbf{x}_t^T$$

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Newton Method – Hessian

- To invert: rewrite the Hessian transformation $\mathbf{C} = \mathcal{H}(\mathbf{B})$ in terms of the source estimates:

$$\mathbf{C} = (\mathbf{B}\mathbf{W}^{-1})^T\mathbf{W}^{-T} + \left\langle \mathrm{diag}\big(f''(\mathbf{y})\big)\mathbf{B}\mathbf{W}^{-1}\mathbf{W}\mathbf{x}\mathbf{y}^T\mathbf{W}^{-T} \right\rangle_N$$

- Define $\tilde{\mathbf{C}} \triangleq \mathbf{C}\mathbf{W}^T$, $\tilde{\mathbf{B}} \triangleq \mathbf{B}\mathbf{W}^{-1}$, $\tilde{\mathbf{C}} = \tilde{\mathcal{H}}(\tilde{\mathbf{B}})$:

$$\tilde{\mathbf{C}} = \tilde{\mathbf{B}}^T + \left\langle \mathrm{diag}\big(f''(\mathbf{y})\big)\tilde{\mathbf{B}}\mathbf{y}\mathbf{y}^T \right\rangle_N$$

- Want to solve linear equation $\mathbf{C} = \mathcal{H}(\mathbf{B})$:

$$\mathbf{B} = \mathcal{H}^{-1}(\mathbf{C}) = \tilde{\mathcal{H}}^{-1}(\mathbf{C}\mathbf{W}^T)\mathbf{W}$$

Newton Method – Hessian

- The Hessian transformation can be simplified using source independence and zero mean:

$$\tilde{c}_{ii} \;\rightarrow\; \tilde{b}_{ii} + E\Big\{ f_i''(y_i) \sum_k \tilde{b}_{ik} y_k y_i \Big\} = \tilde{b}_{ii}(1+\eta_i)$$

$$\tilde{c}_{ij} \;\rightarrow\; \tilde{b}_{ji} + E\Big\{ f_i''(y_i) \sum_k \tilde{b}_{ik} y_k y_j \Big\} = \tilde{b}_{ji} + \kappa_i \sigma_j^2 \tilde{b}_{ij}$$

$$\tilde{c}_{ji} \;\rightarrow\; \tilde{b}_{ij} + E\Big\{ f_j''(y_j) \sum_k \tilde{b}_{jk} y_k y_i \Big\} = \tilde{b}_{ij} + \kappa_j \sigma_i^2 \tilde{b}_{ji}$$

$$\eta_i \triangleq E\{y_i^2 f_i''(y_i)\}, \quad \kappa_i \triangleq E\{f_i''(y_i)\}, \quad \sigma_i^2 \triangleq E\{y_i^2\}$$

- This leads to 2x2 block diagonal form:

$$\begin{bmatrix} \tilde{c}_{ij} \\ \tilde{c}_{ji} \end{bmatrix} = \begin{bmatrix} \kappa_i \sigma_j^2 & 1 \\ 1 & \kappa_j \sigma_i^2 \end{bmatrix} \begin{bmatrix} \tilde{b}_{ij} \\ \tilde{b}_{ji} \end{bmatrix}$$

Newton Direction

- Invert Hessian transformation, evaluate at gradient:

$$\Delta \mathbf{W} = \tilde{\mathcal{H}}^{-1}(-\mathbf{G}\mathbf{W}^T)\mathbf{W}$$

- Leads to the following equations:

$$\tilde{\mathbf{B}} = \tilde{\mathcal{H}}^{-1}(-\mathbf{G}\mathbf{W}^T)$$

$$\mathbf{\Phi} \triangleq \frac{1}{N}\sum_{t=1}^{N} \mathbf{g}_t \mathbf{y}_t^T$$

$$-\mathbf{G}\mathbf{W}^T = \mathbf{I} - \mathbf{\Phi}$$

$$\begin{aligned} \tilde{b}_{ii} &= \frac{1-\phi_{ii}}{1+\eta_i}, \quad i = 1, \ldots, n \\ \tilde{b}_{ij} &= \frac{\phi_{ji} - \kappa_j \sigma_i^2 \phi_{ij}}{\kappa_i \kappa_j \sigma_i^2 \sigma_j^2 - 1}, \quad \forall i \neq j \end{aligned}$$

- Calculate the Newton direction:

$$\Delta \mathbf{W} = \tilde{\mathbf{B}}\mathbf{W}$$

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Positive Definiteness of Hessian

- Conditions for positive definiteness:

$$
\begin{aligned}
&1)\quad 1+\eta_i > 0, \quad \forall i \\
&2)\quad \kappa_i > 0, \quad \forall i, \text{ and,} \\
&3)\quad \kappa_i \kappa_j \sigma_i^2 \sigma_j^2 - 1 > 0, \quad \forall i \neq j
\end{aligned}
$$

- Always true for true when model source densities match true densities:

1)

$$
\begin{aligned}
1 + E\{y^2 f''(y)\} &= \int_{-\infty}^{\infty} \left(y^2 f'(y)^2 - 2yf'(y) + 1\right) p(y)\,dy \\
&= E\left\{\left(yf'(y) - 1\right)^2\right\} \geq 0
\end{aligned}
$$

2)

$$
E\{f''(y)\} = \int_{-\infty}^{\infty} f'(y)^2 p(y)\,dy = E\left\{f'(y)^2\right\} > 0
$$

3)

$$
E\{y^2\}E\{f''(y)\} = E\left\{y^2\right\}E\left\{f'(y)^2\right\} \geq \left(E\{yf'(y)\}\right)^2 = 1
$$

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Newton for ICA Mixture Model

- Similar derivation applies to ICA mixture model:

$$p(\mathbf{X};\Theta) = \sum_{\mathbf{V},\mathbf{Z}} \prod_{t=1}^{N} \prod_{h=1}^{M} \gamma_h^{v_{ht}} |\det \mathbf{W}_h|^{v_{ht}} \prod_{i=1}^{n} \prod_{j=1}^{m} {Q_{hijt}^{l}}^{v_{ht} z_{hijt}}$$

$$F^l(\Theta) = \sum_{t=1}^{N} \sum_{h=1}^{M} \Big[\hat{v}_{ht}^l \big(-\log \gamma_h - \log|\det \mathbf{W}_h| \big) + \sum_{i=1}^{n} \sum_{j=1}^{m} \hat{r}_{hijt}^l \big(-\log \alpha_{hij} - \tfrac{1}{2}\log \beta_{hij} + f_{hij}(y_{hijt}) \big) \Big]$$

$$\mathbf{C} = \mathbf{W}_h^{-T} \mathbf{B}^T \mathbf{W}_h^{-T} + \frac{1}{\sum_t \hat{v}_{ht}^l} \sum_{t=1}^{N} \mathbf{D}_{ht}^l \mathbf{B} (\mathbf{x}_t - \mathbf{c}_h)(\mathbf{x}_t - \mathbf{c}_h)^T$$

$$\tilde{\mathbf{C}} = \tilde{\mathbf{B}}^T + \frac{1}{\sum_t \hat{v}_{ht}^l} \sum_{t=1}^{N} \mathbf{D}_{ht}^l \tilde{\mathbf{B}} \mathbf{y}_{ht} \mathbf{y}_{ht}^T$$

$$\mathbf{y}_{ht} \triangleq \mathbf{W}_h (\mathbf{x}_t - \mathbf{c}_h)$$

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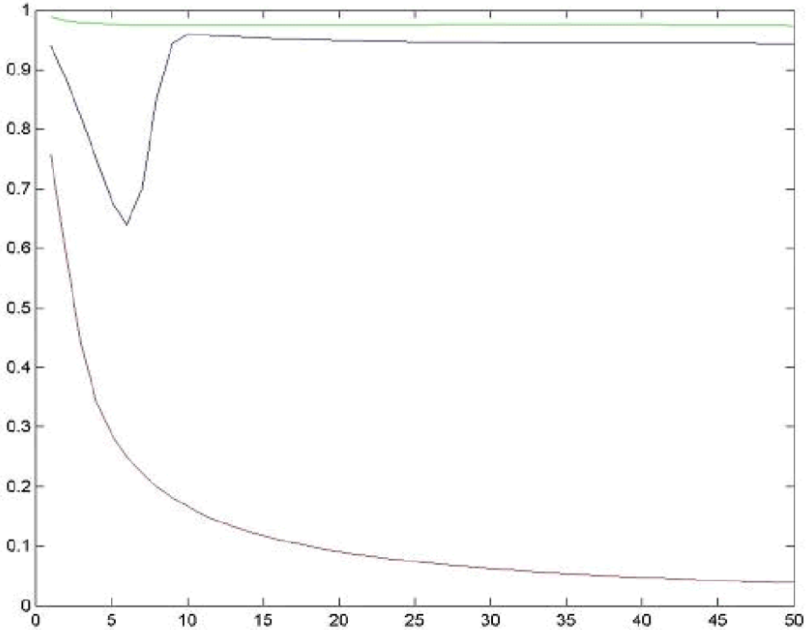
Convergence Rates

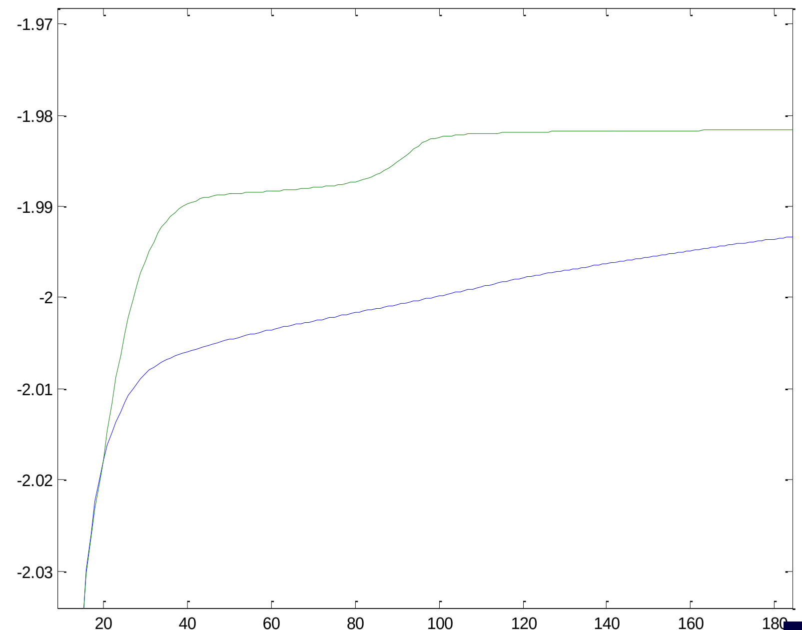
- Convergence is really much faster than natural gradient. Works with step size 1.0!
- Need correct source density model

$$\|\mathbf{W}^{l+1} - \mathbf{W}^*\| \,/\, \|\mathbf{W}^l - \mathbf{W}^*\|$$



iteration

log likelihood



iteration

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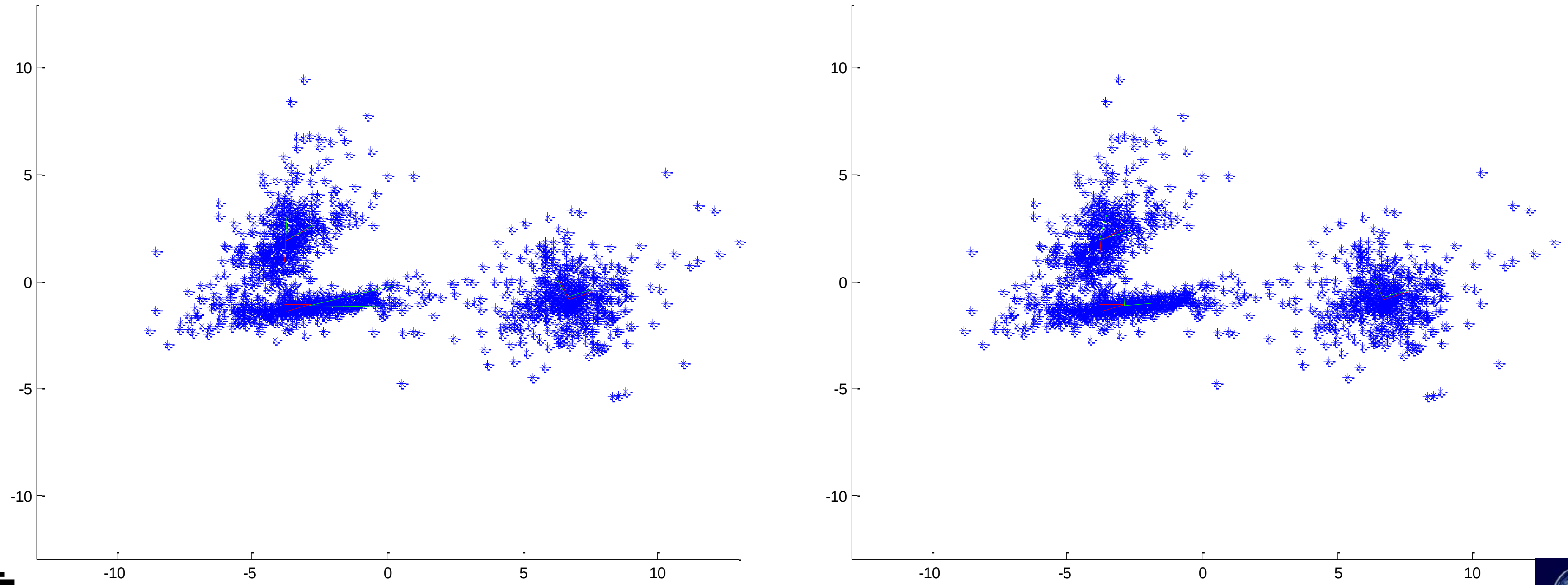
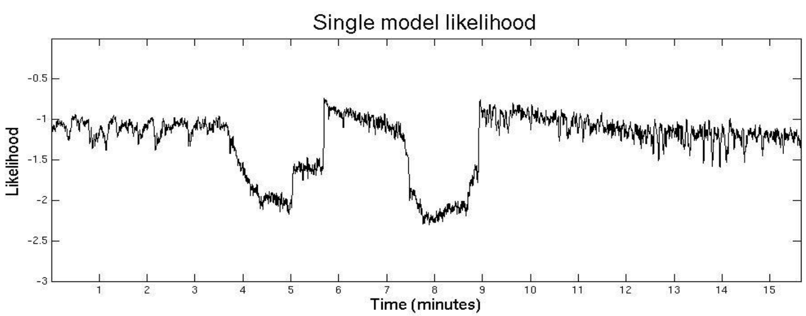
Natural Gradient Vs. Newton

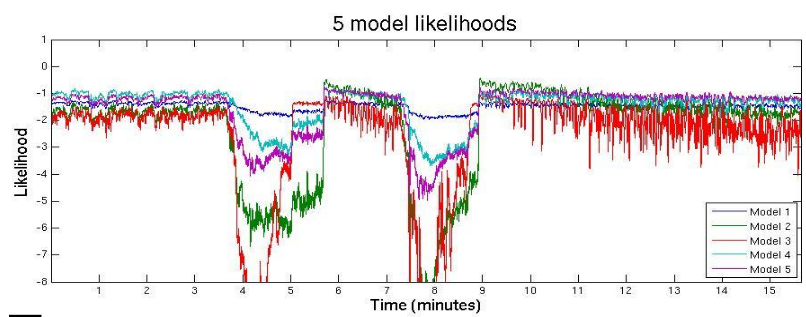
- 3 models in two dimensions, 500 pts per model
- Newton method converges, natural gradient (Lee et al.) is slow or fails to converge, has difficulty on poorly conditioned models

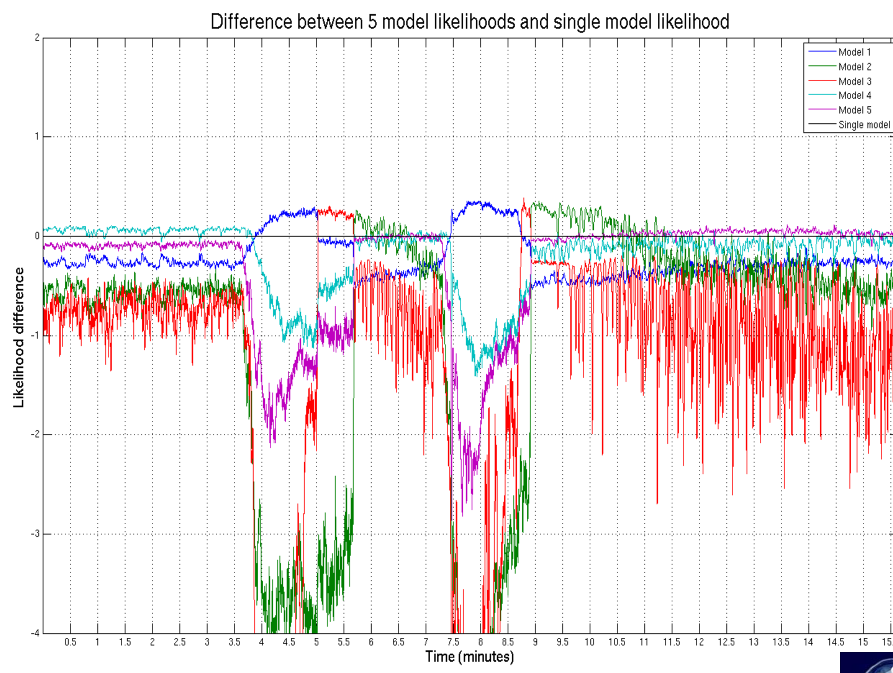


Epilepsy

- Data: 15 minutes from 1 subject containing 2 seizures
- Single model does not represent seizure well
- We learned 5 models – new models consistently adapt to portions of seizure

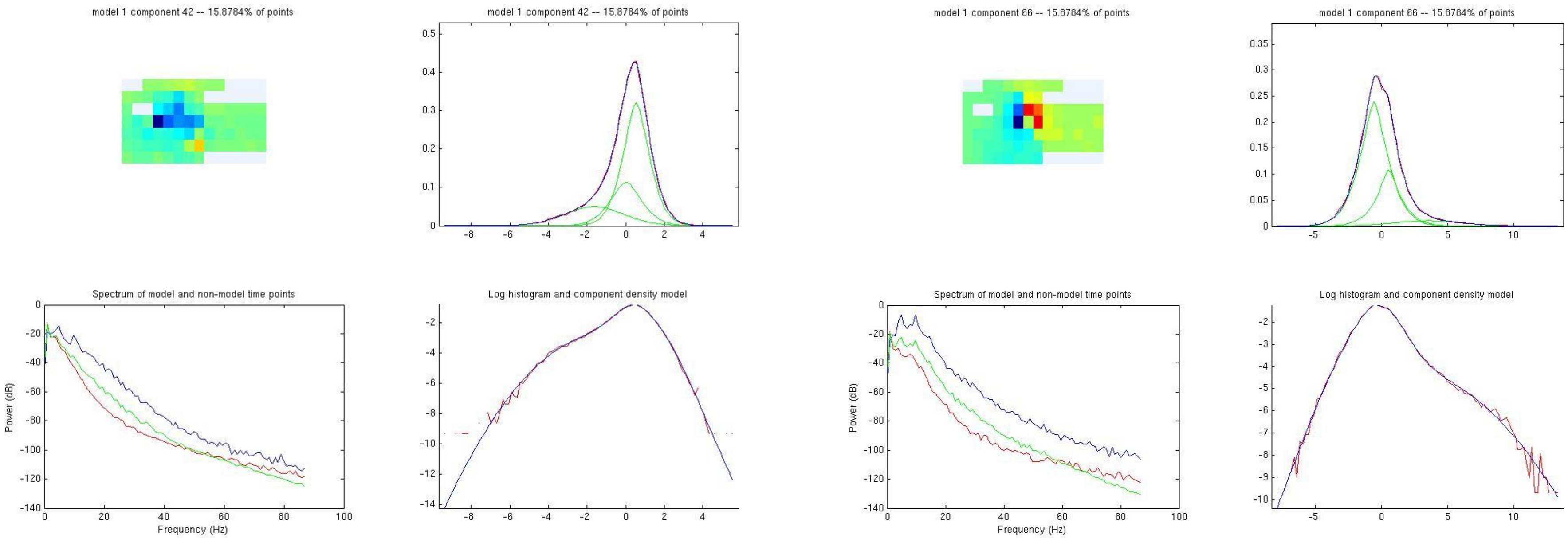




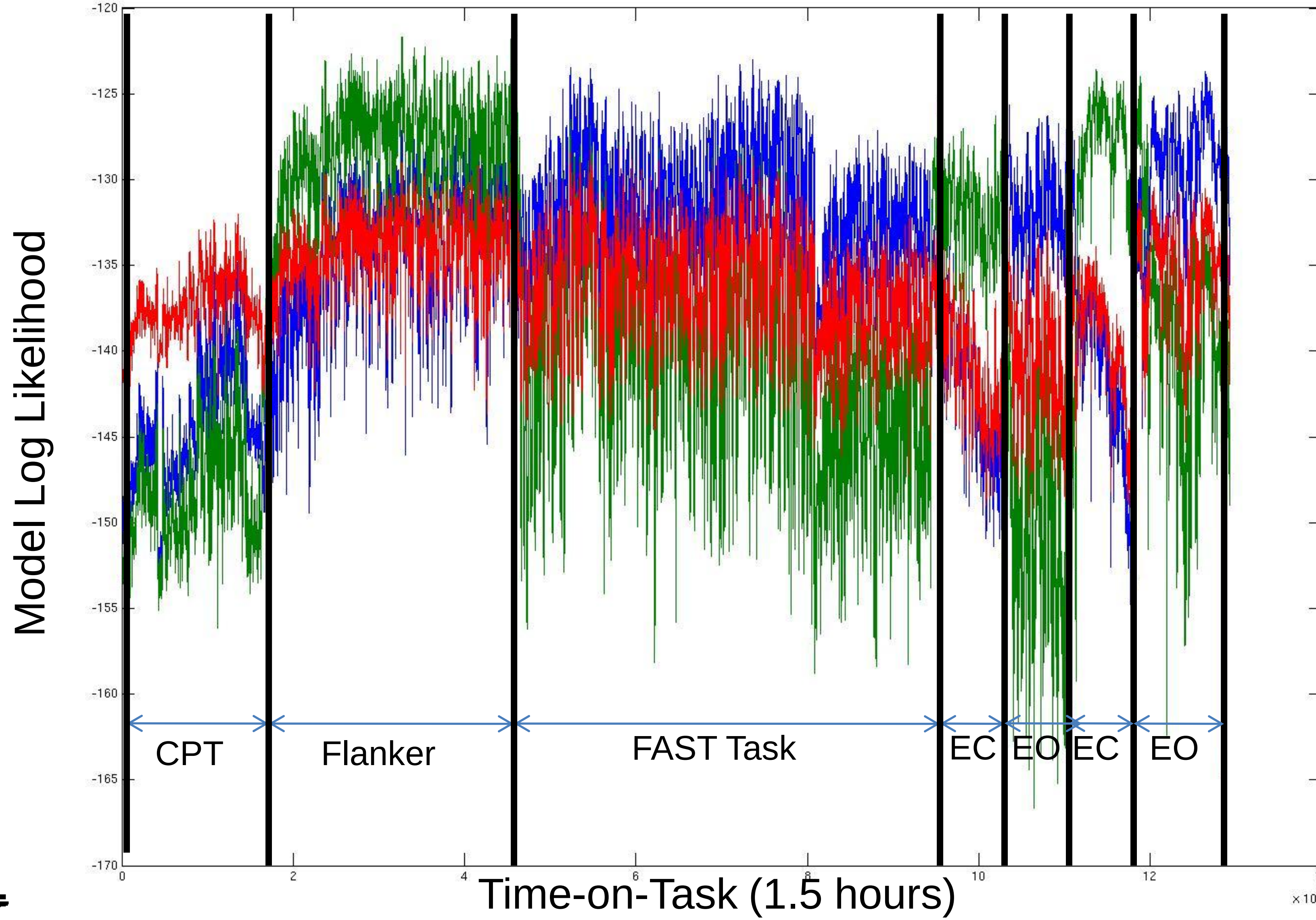


Epilepsy Grid Maps

- Maps from grid of electrodes placed intercranially over seizure area
- Source probability densities are fit by mixture model



Segmentation of Tasks



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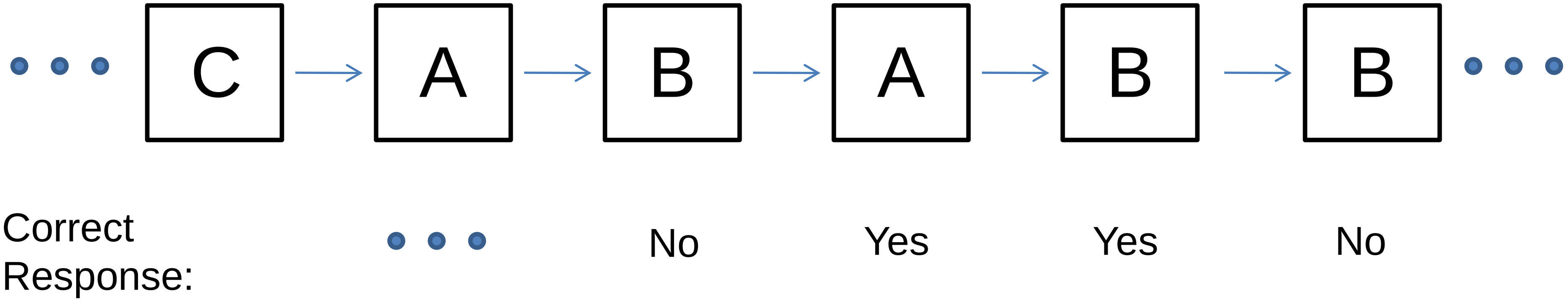
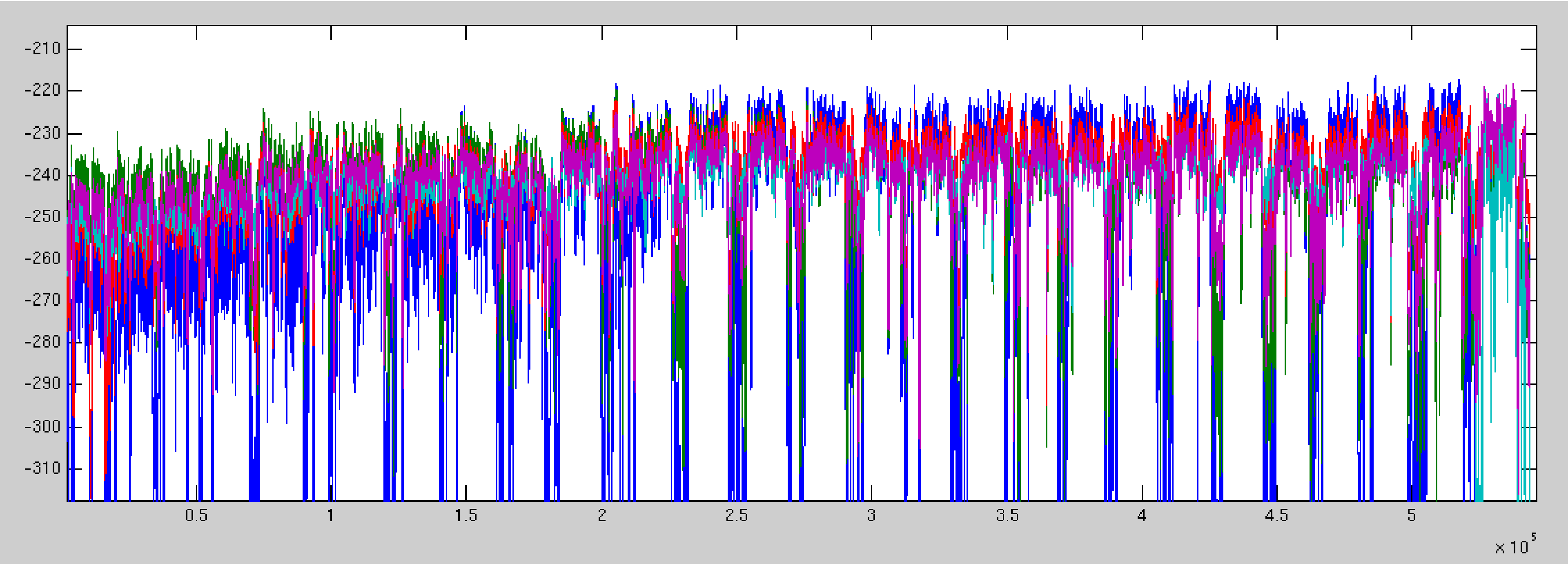
Twoback Task

- Data recording supervised by Julie Onton
- Subject presented with sequence of letters and must respond whether current letter is the same as the one two letters back



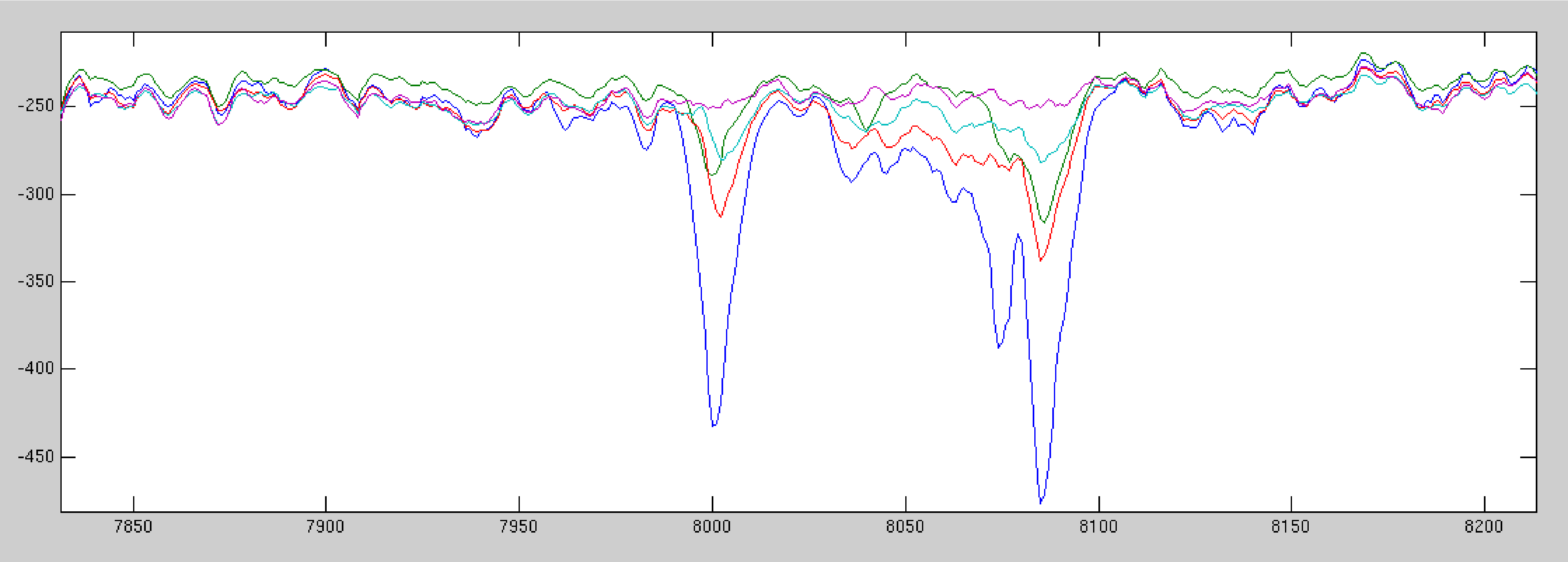
bt73 segmentation

- Task trials are represented by green and blue models
- Inter-task intervals represented by red and cyan model
- Eye blinks represented by magenta model



bt73 segmentation zoom (green)

- Task trials are represented by green and blue models
- Inter-task intervals represented by red and cyan model
- Eye blinks represented by magenta model



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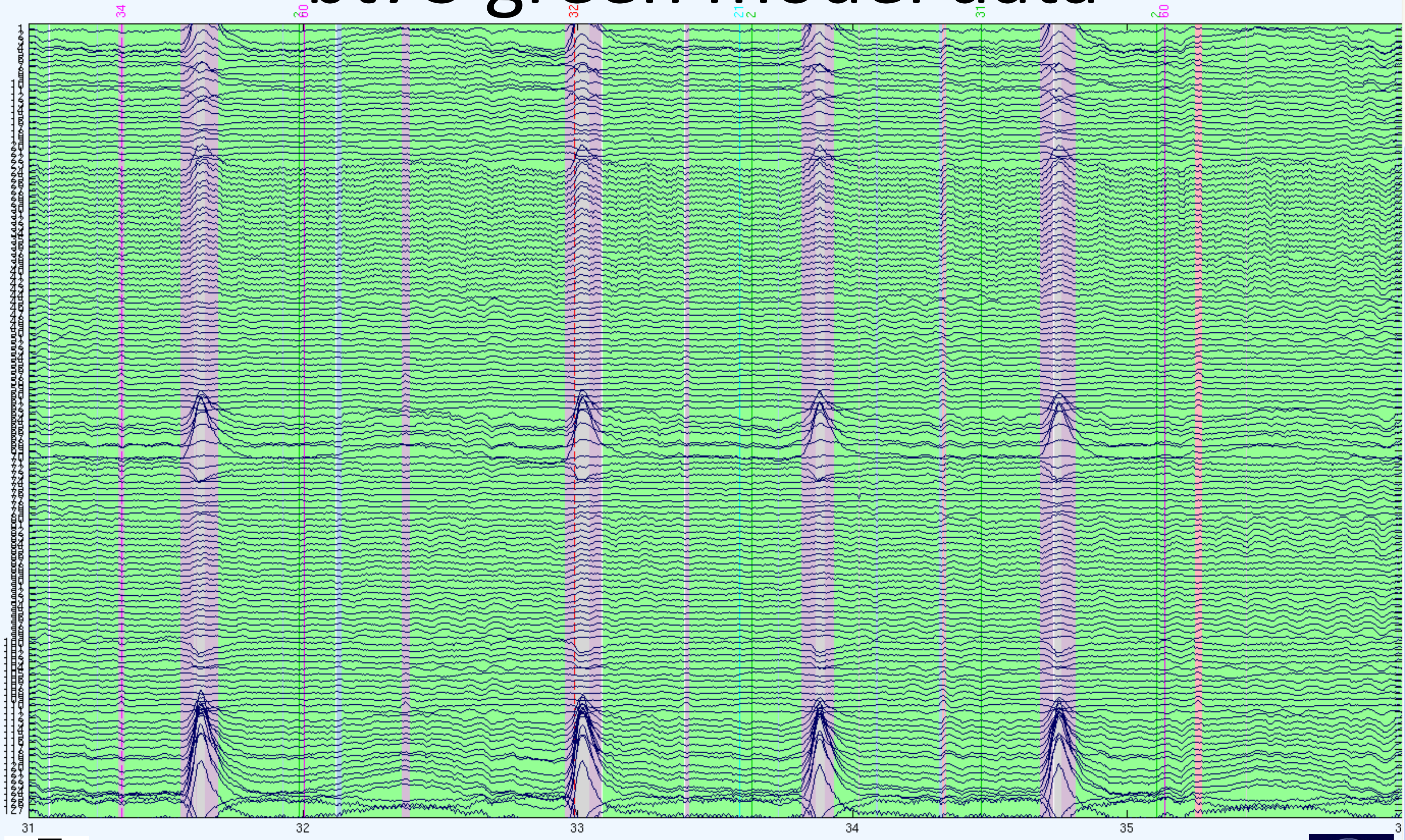
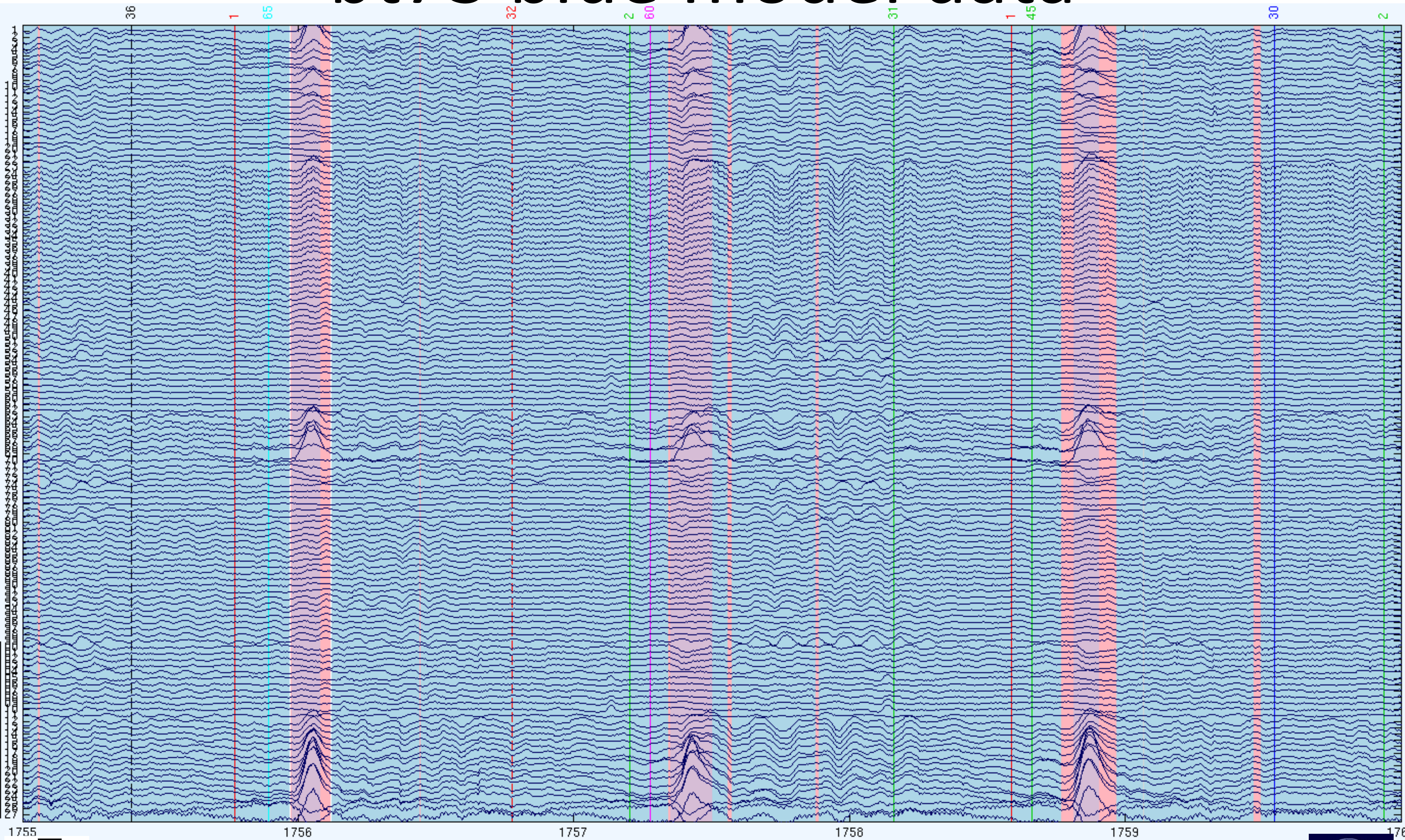
bt73 green model data



bt73 blue model data



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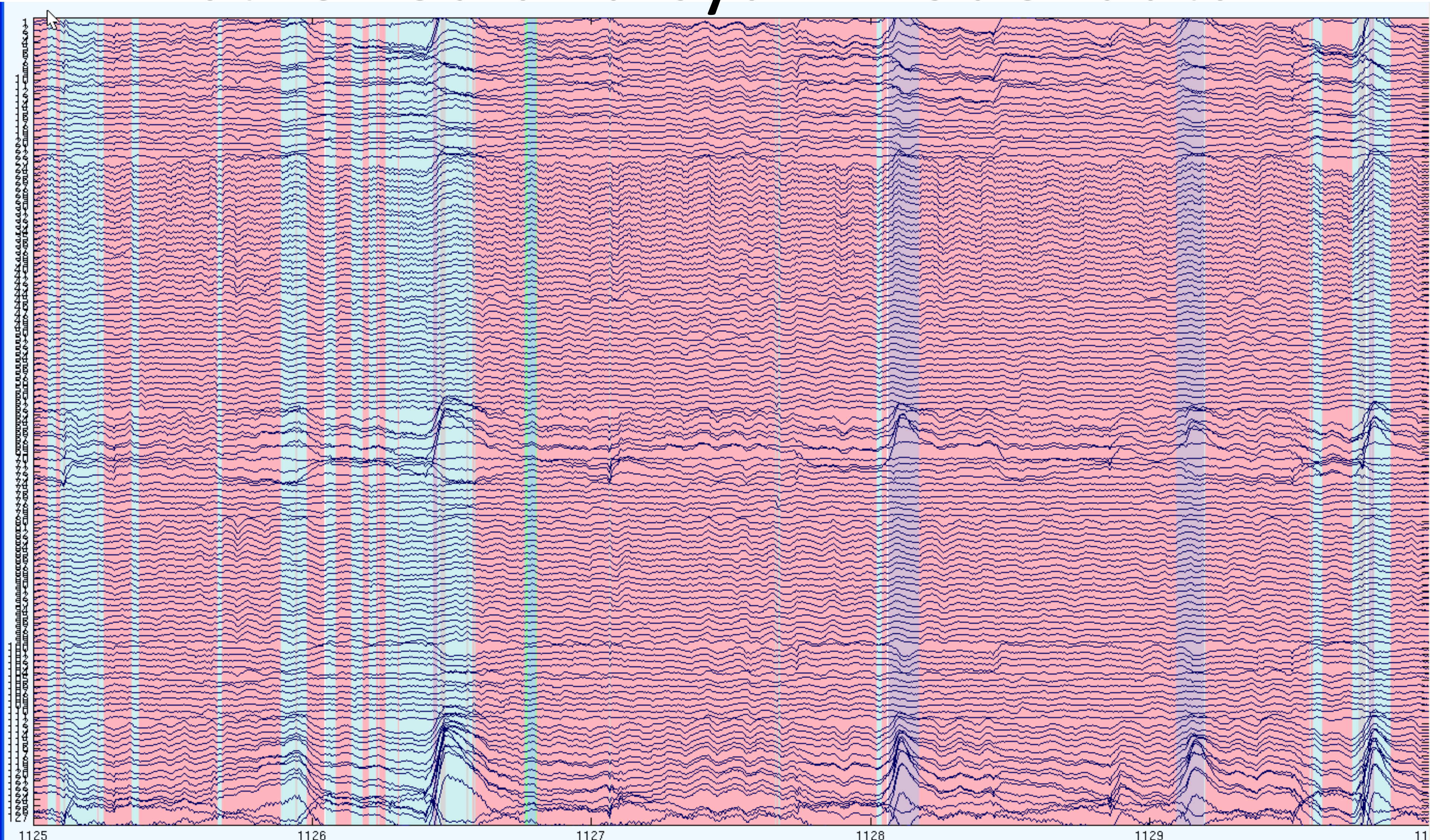
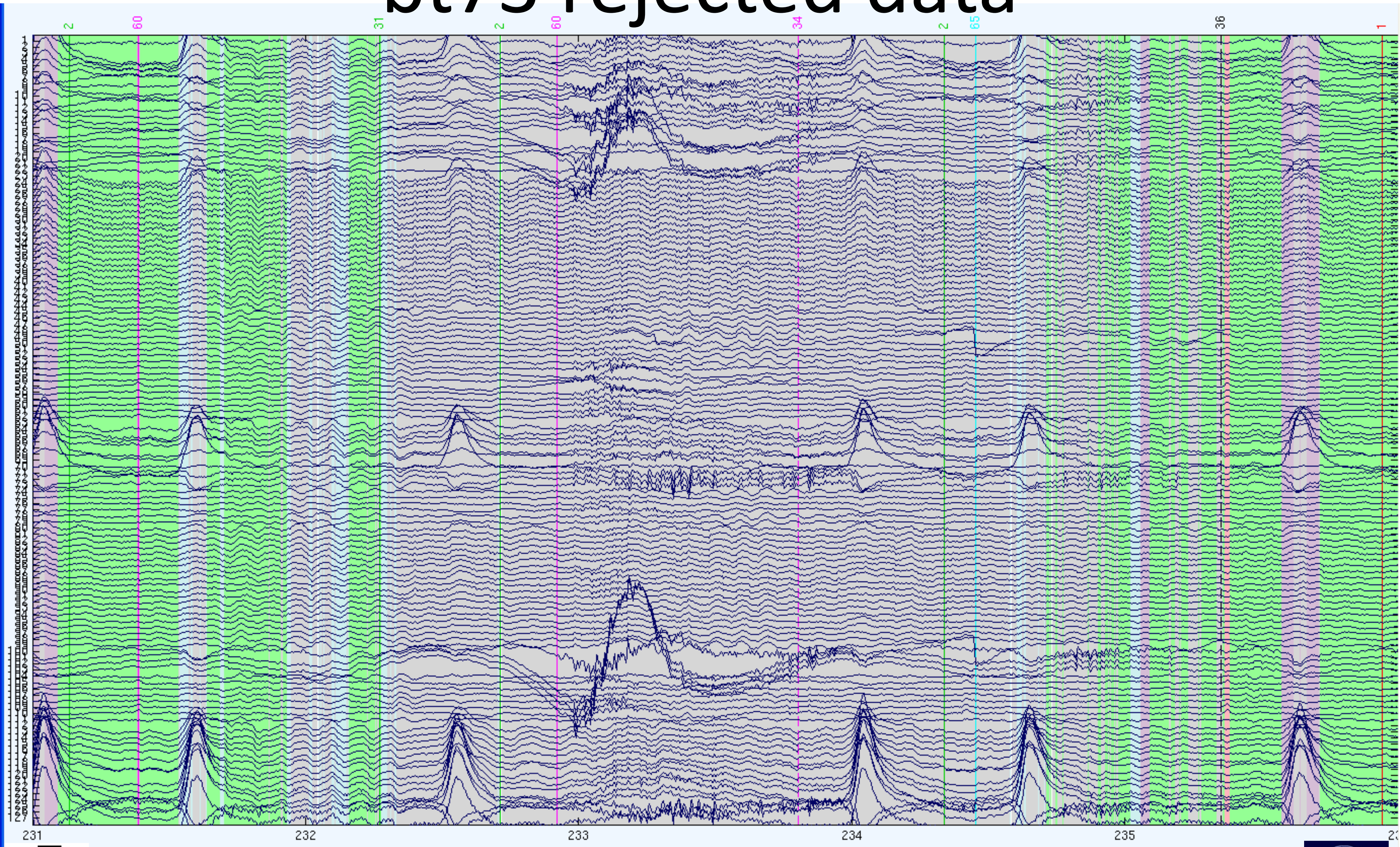
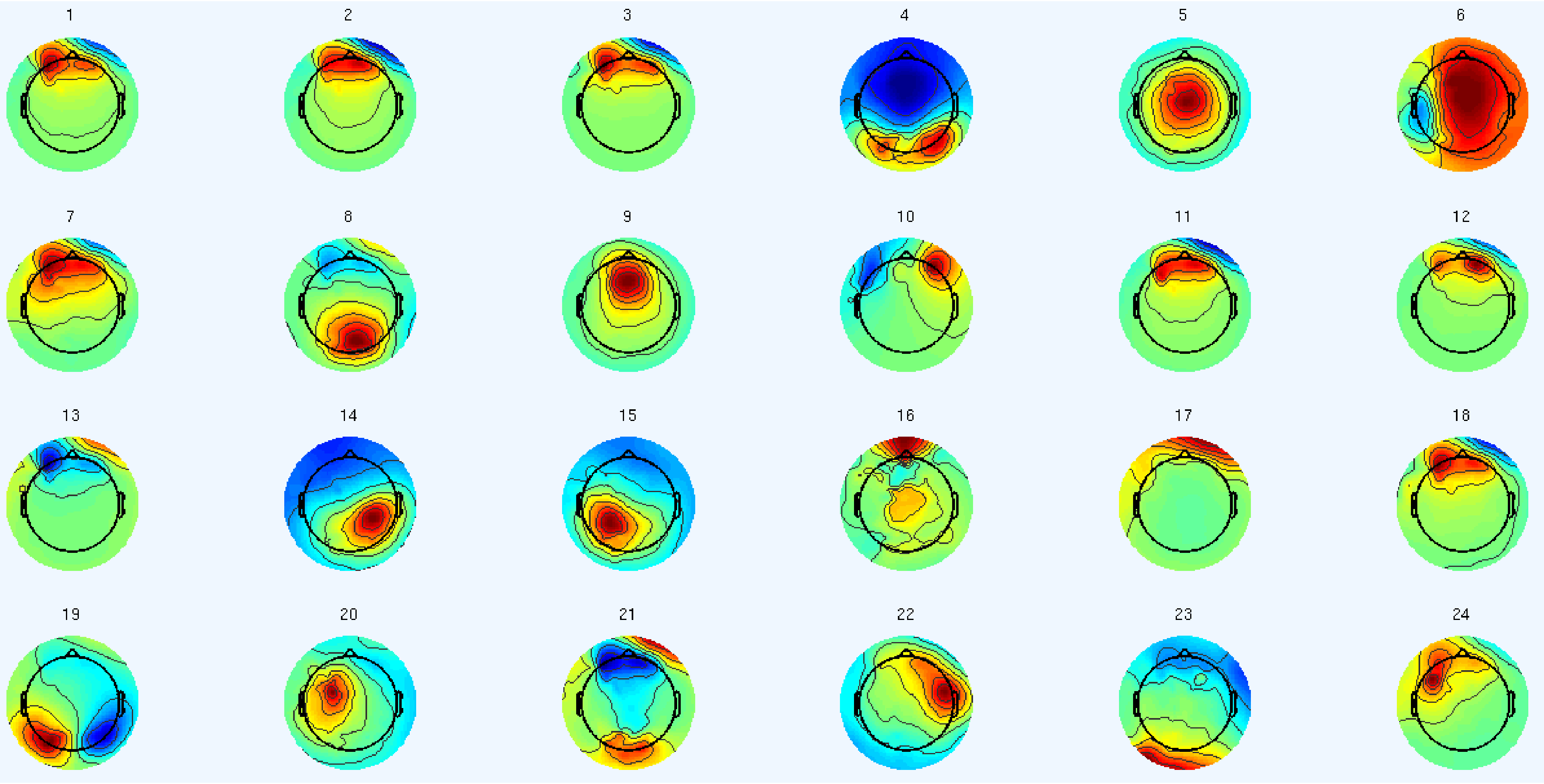
bt73 red and cyan model data



bt73 rejected data

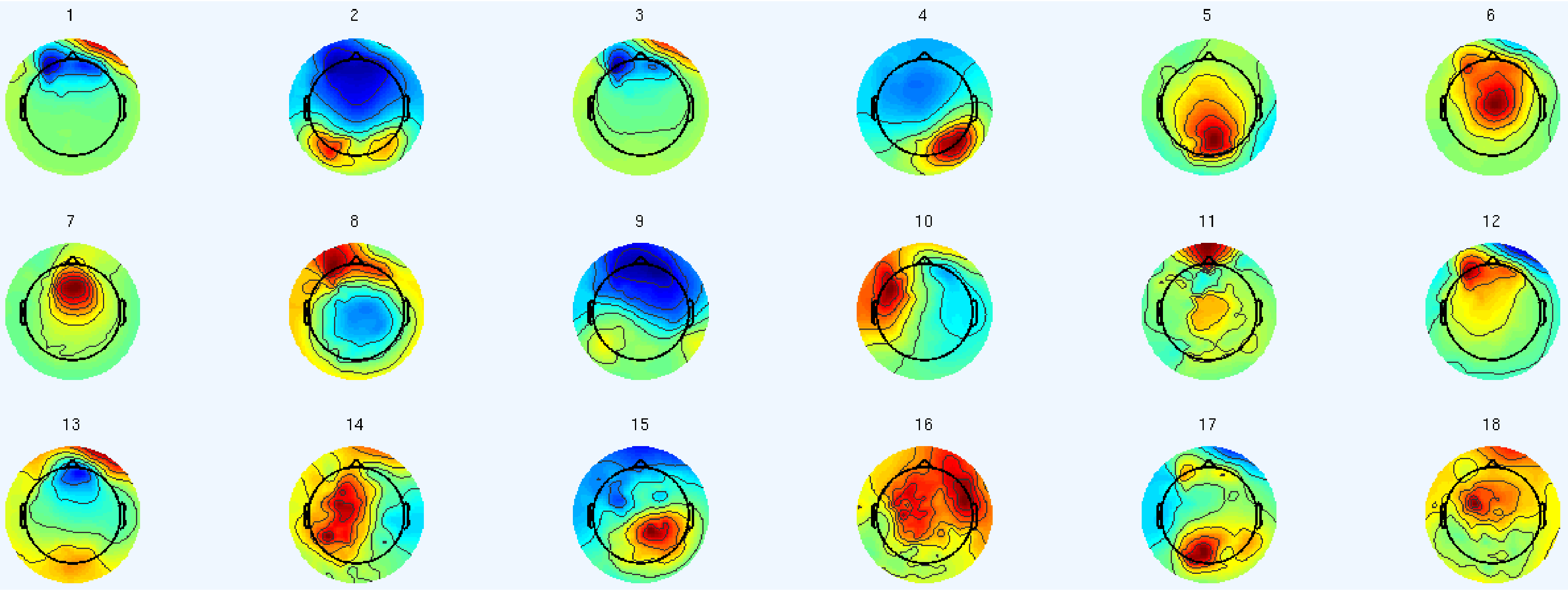


bt73 single model components



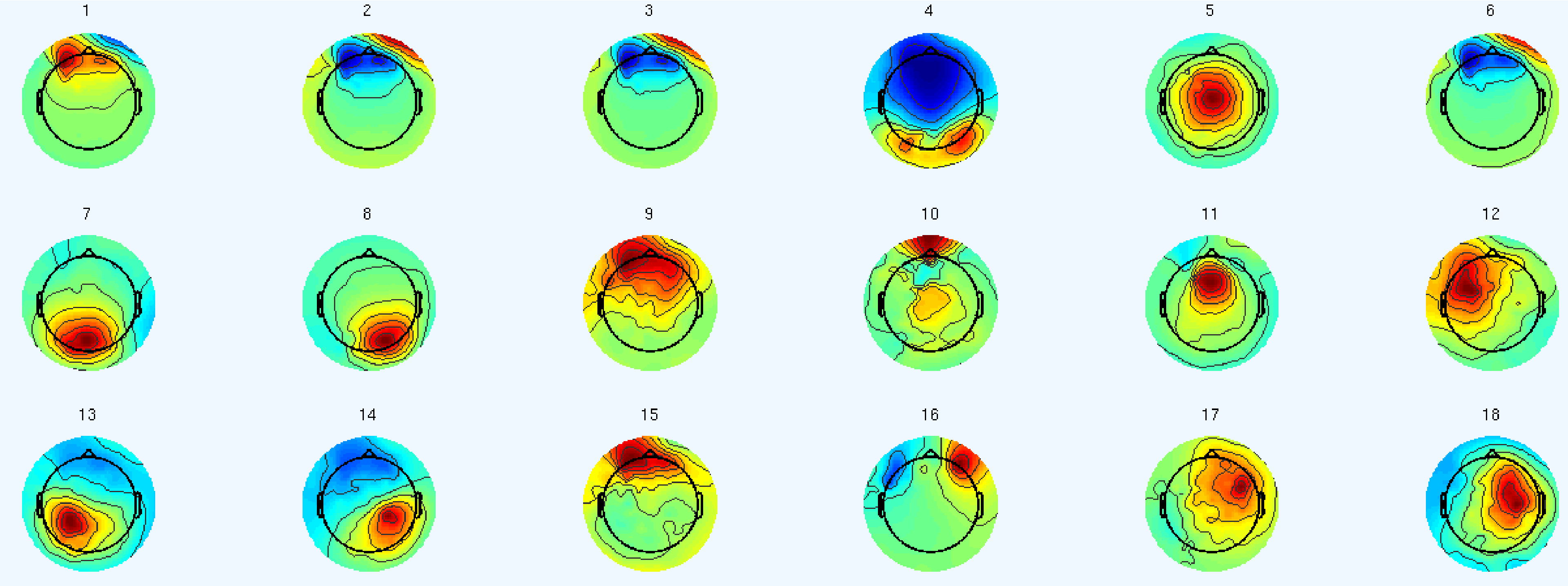
bt73 green model components

- Prominent alpha and frontal midline components
- Weak mu components



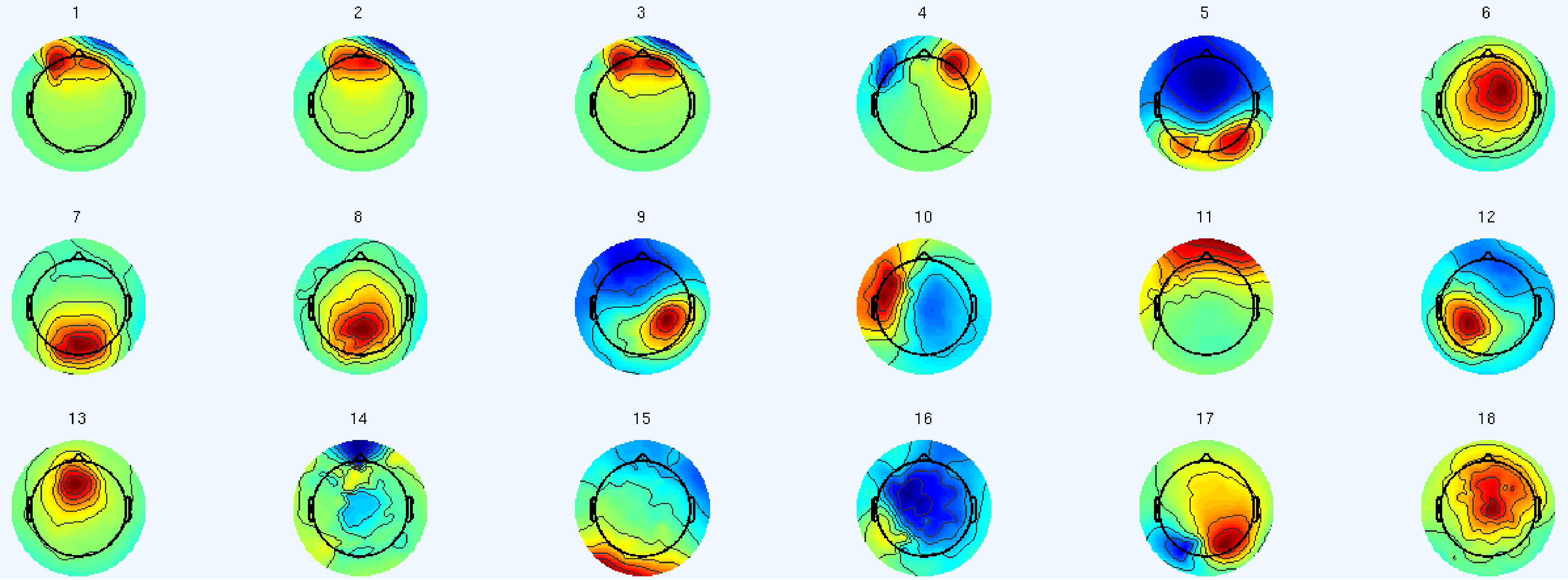
bt73 blue model components

- Prominent alpha and central midline components
- Weak mu components
- Different occipital alpha components (7, 8)



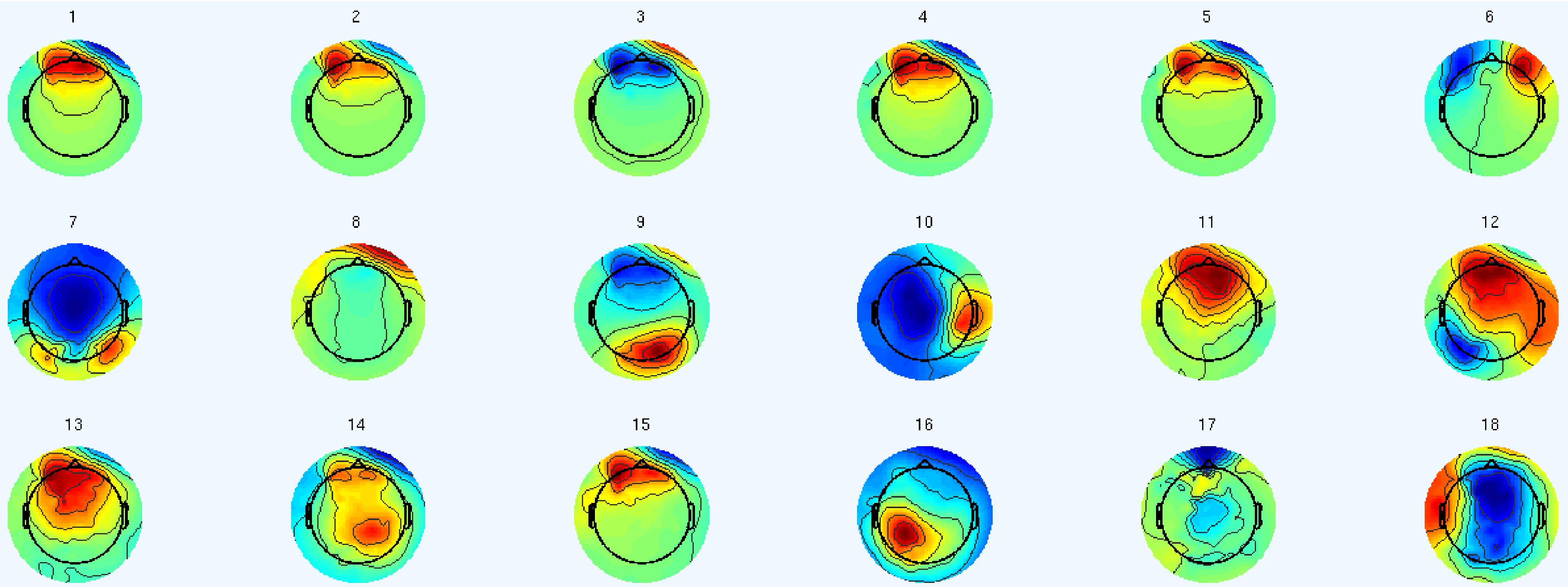
bt73 red model components

- Prominent alpha and central midline components
- Lateral eye movement component (4)
- Tangential occipital component (17)



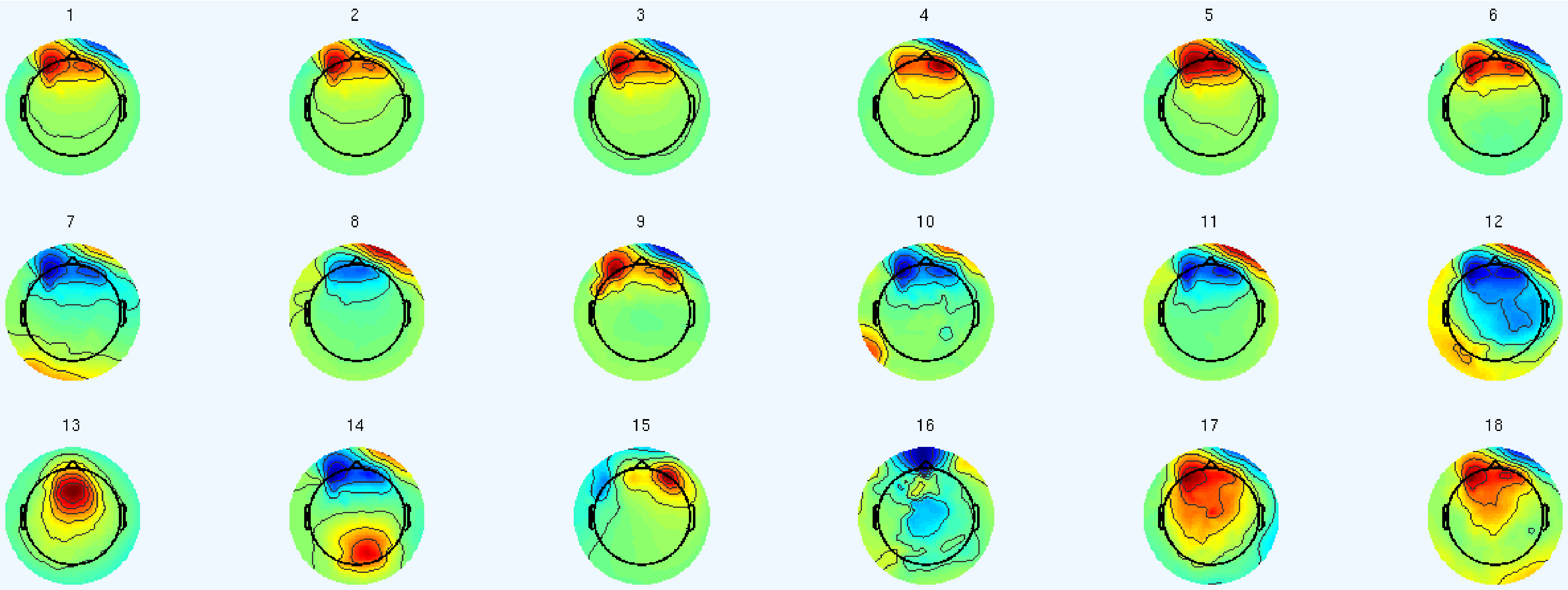
bt73 cyan model components

- Prominent eye-blink components (1-5)
- Lateral eye-movement (6)



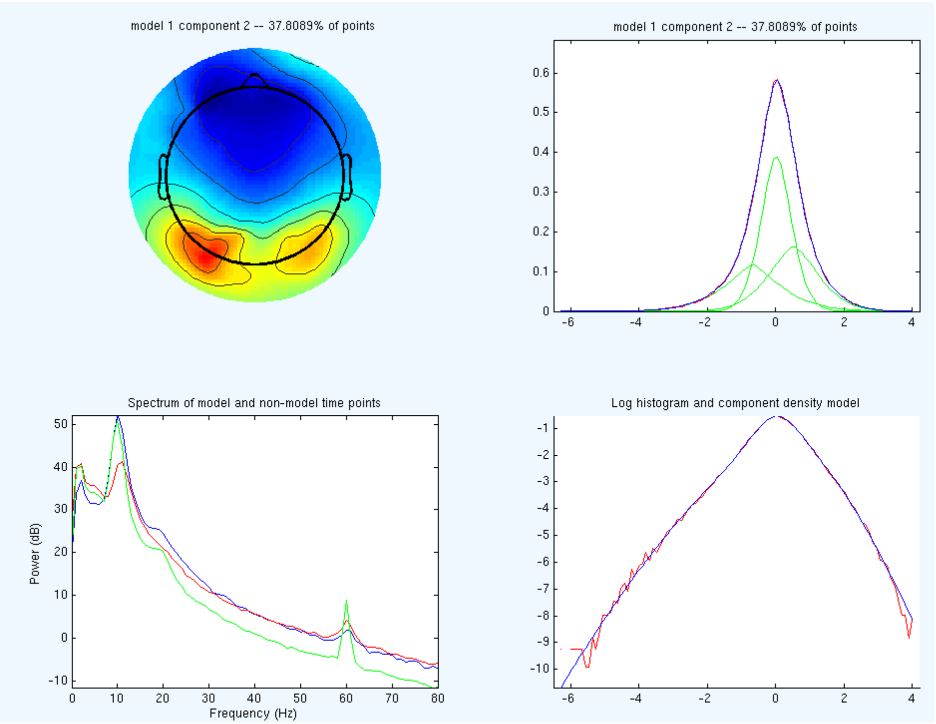
bt73 magenta model components

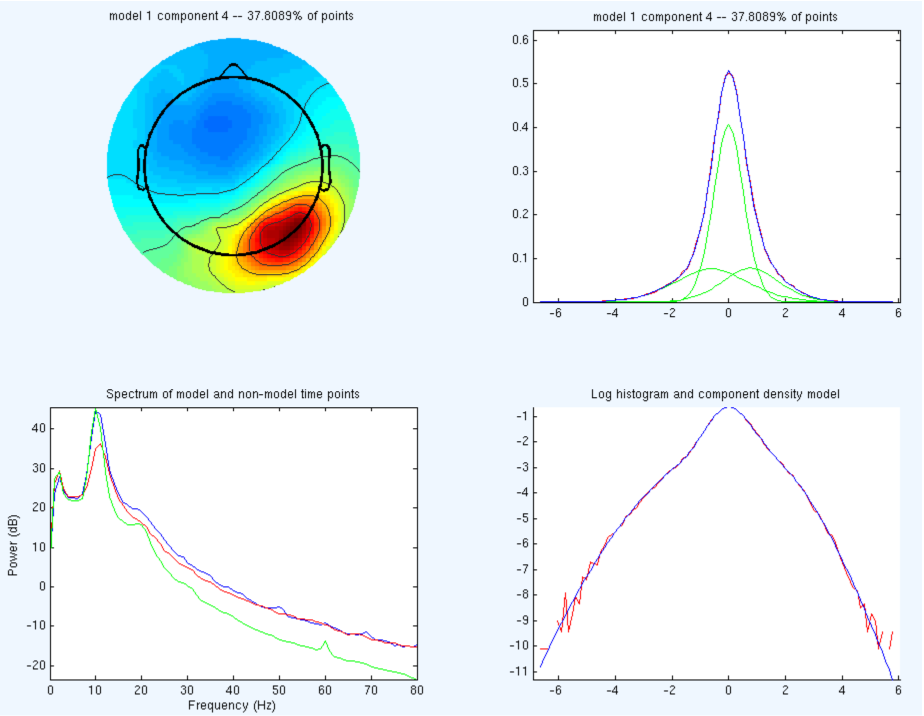
- Mostly eye-blink components (1-12)
- Frontal midline component (13)



bt73 green model alpha

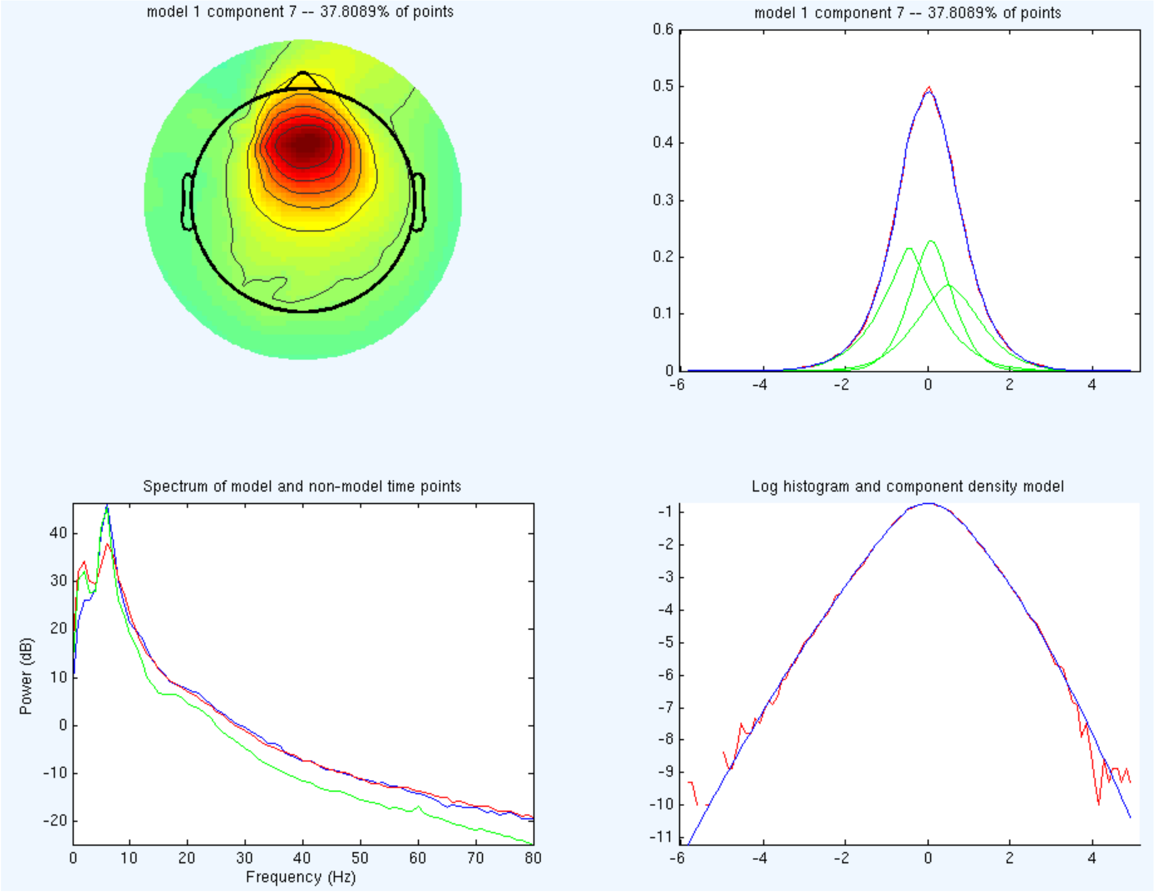
- Components have more power in segments represented by model than in non-model segments





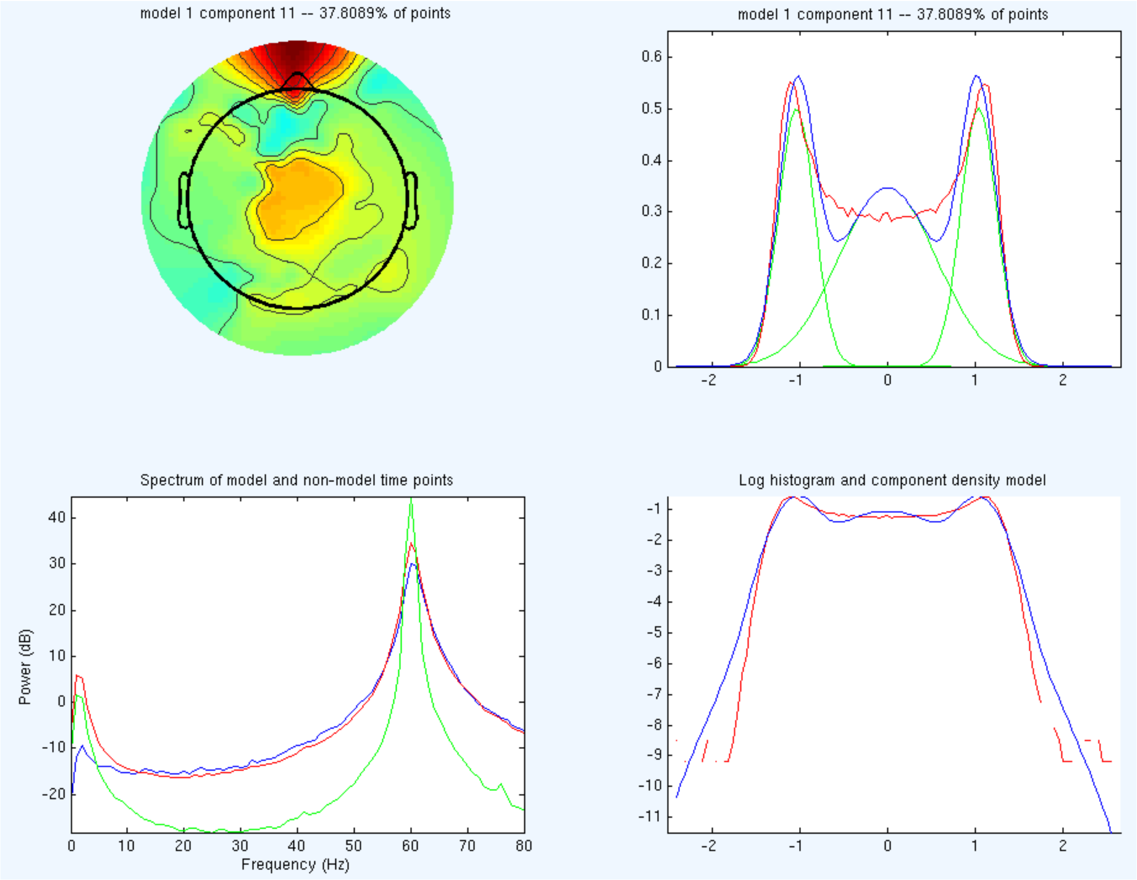
bt73 green model frontal midline θ

- Component again has more power in segments represented by model than in non-model segments



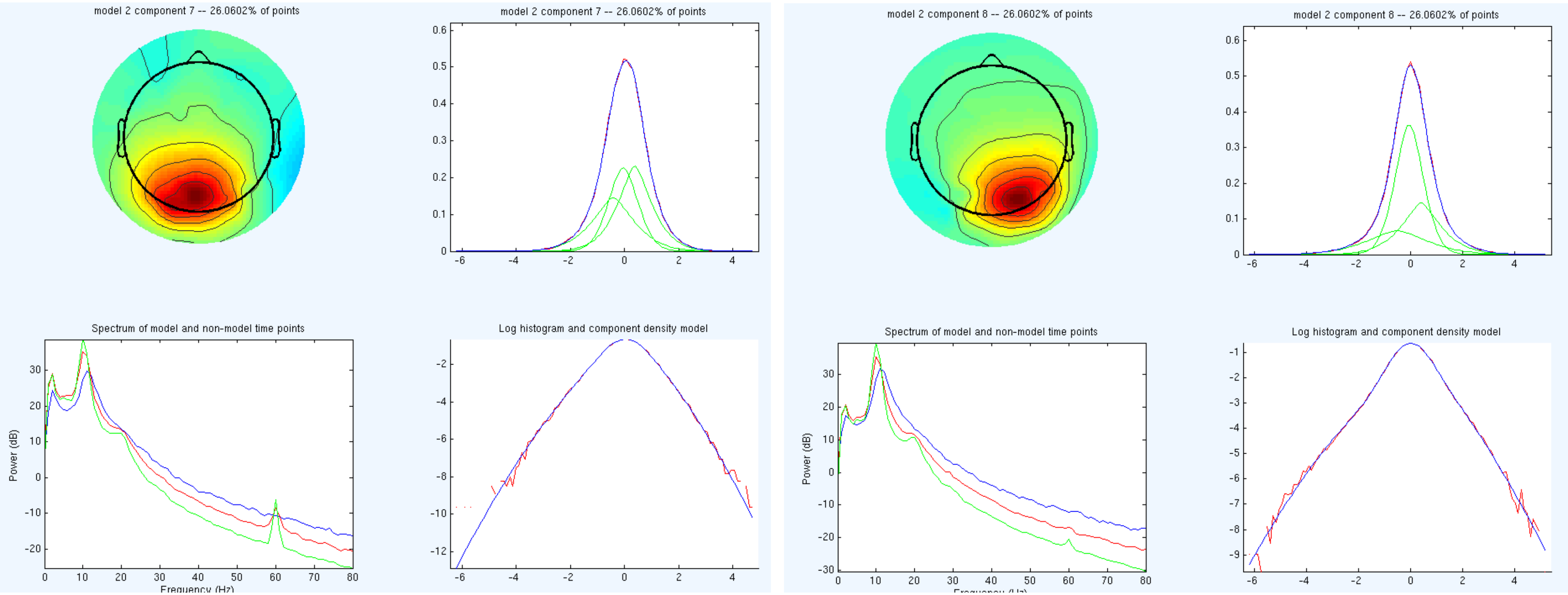
bt73 green model power line comp

- Sub-Gaussian component represented by mixture model of Generalized Gaussian densities



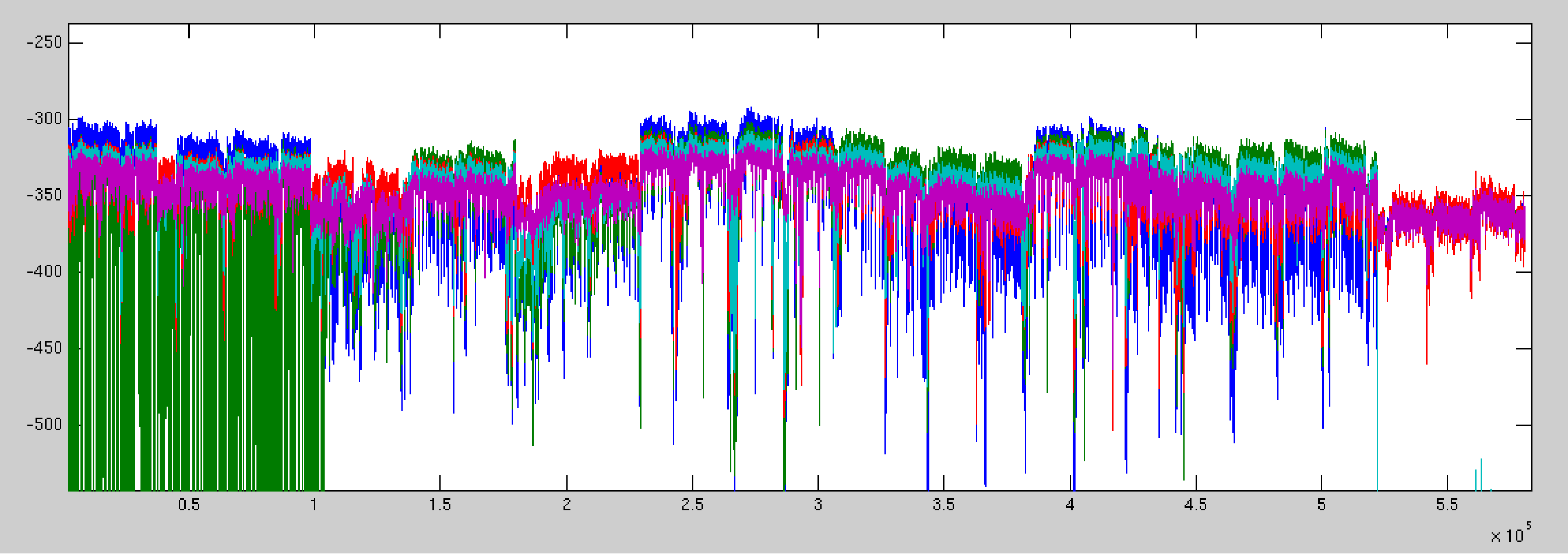
bt73 blue model alpha

- Alpha peak shifted in model segments shifted slightly higher than in non-model time segments

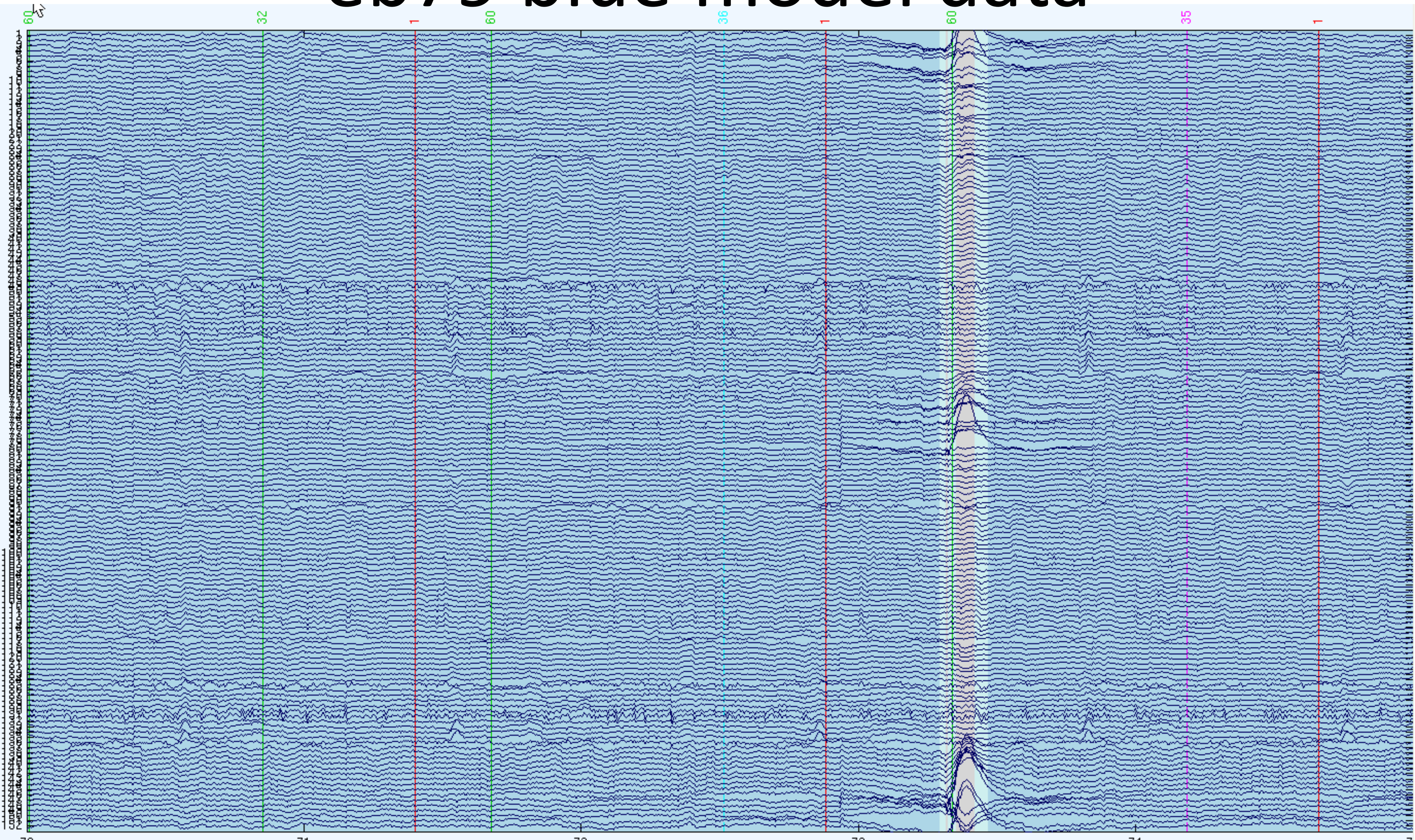


eb79 segmentation

- Task trials are represented by blue, green, and red models
- Red model contains muscle activity not present in blue and green
- Eye blinks represented by cyan and magenta models



eb79 blue model data



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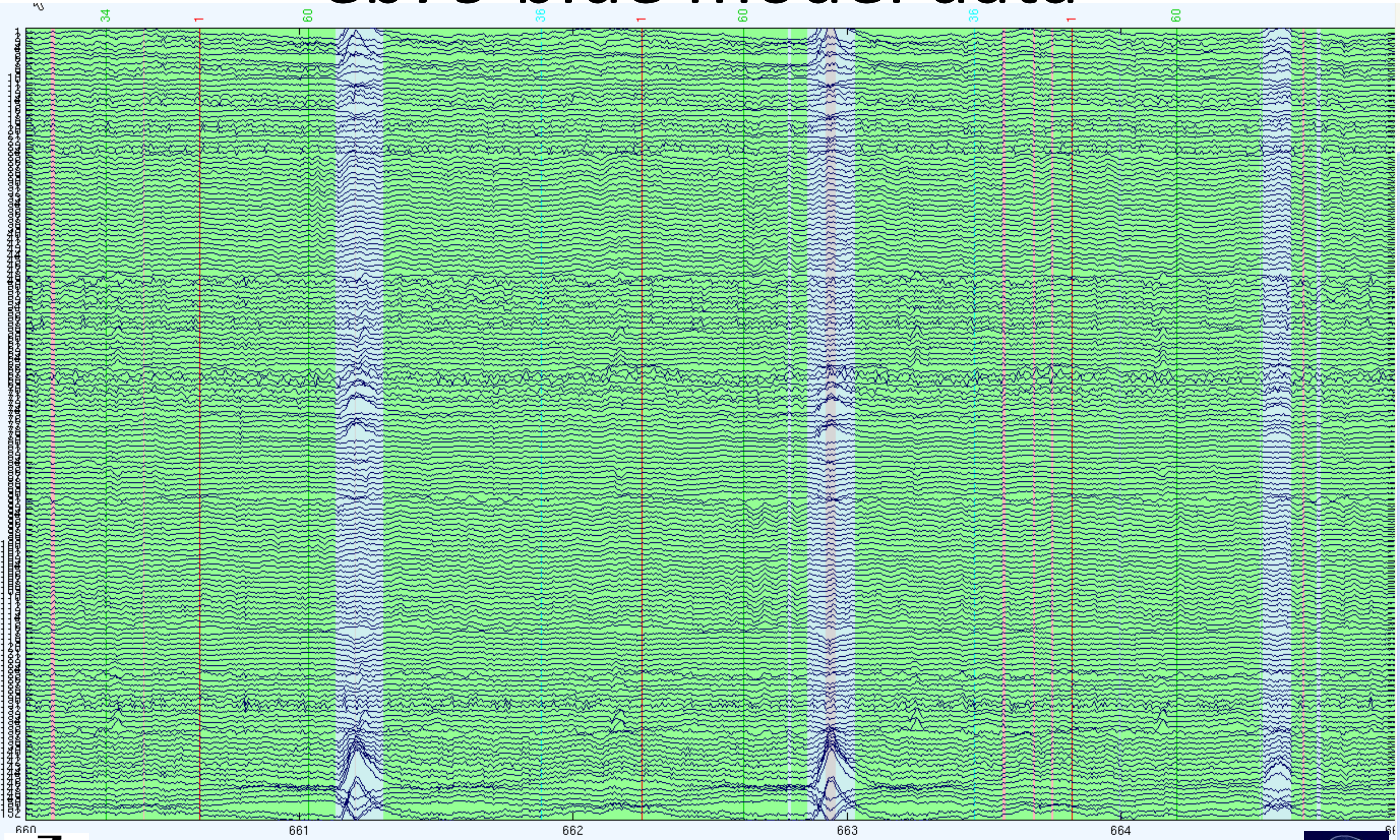

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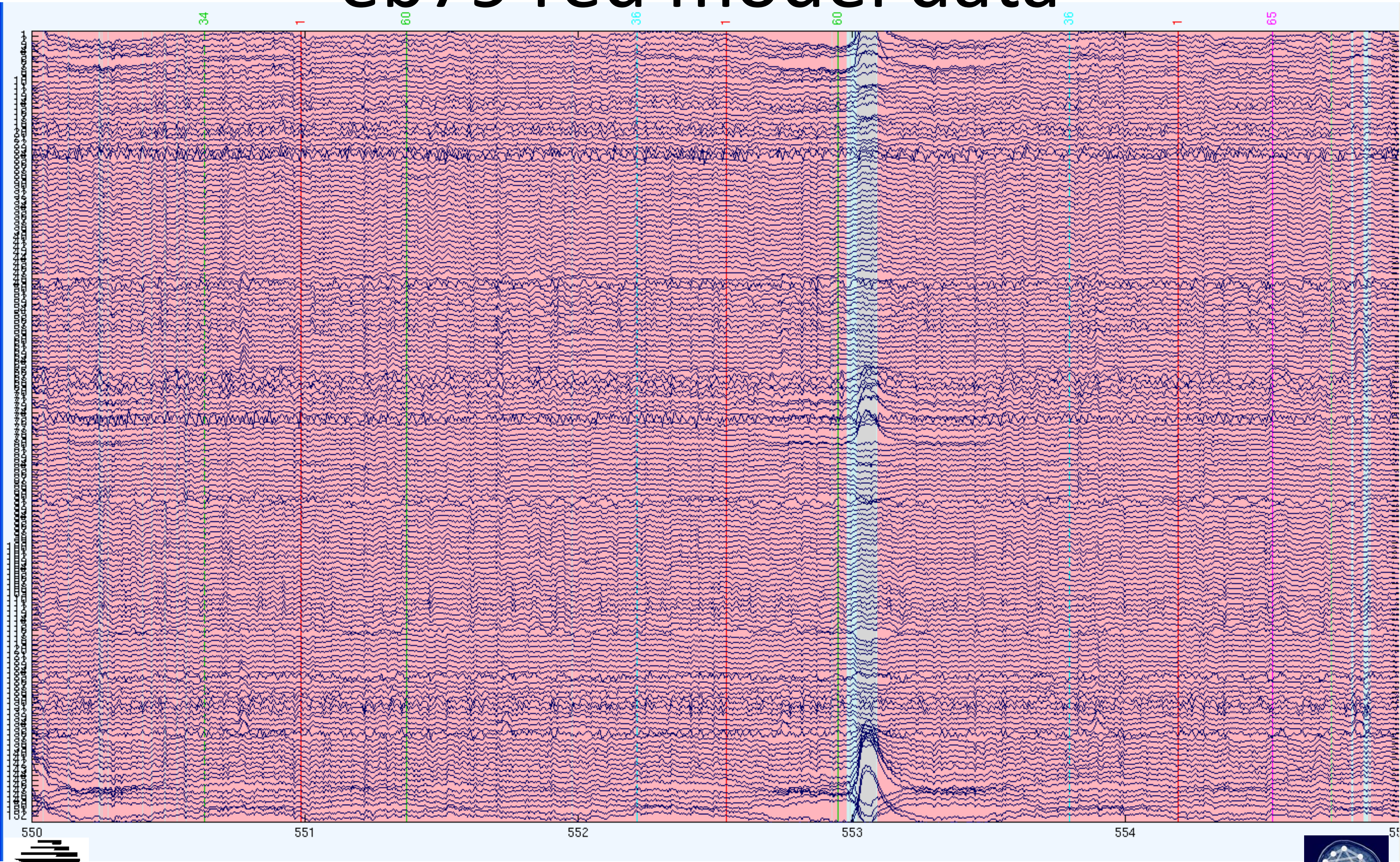
eb79 red model data



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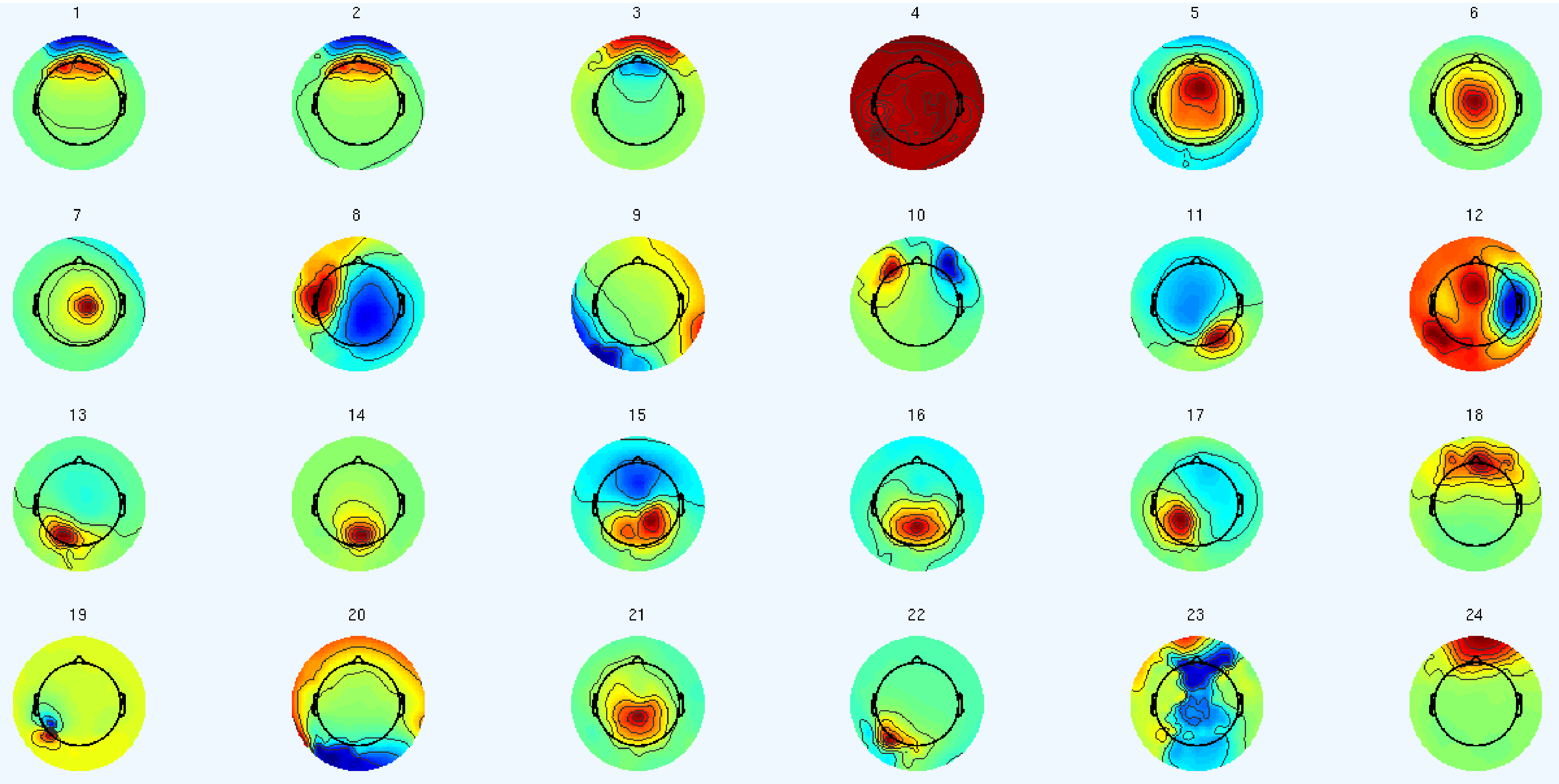
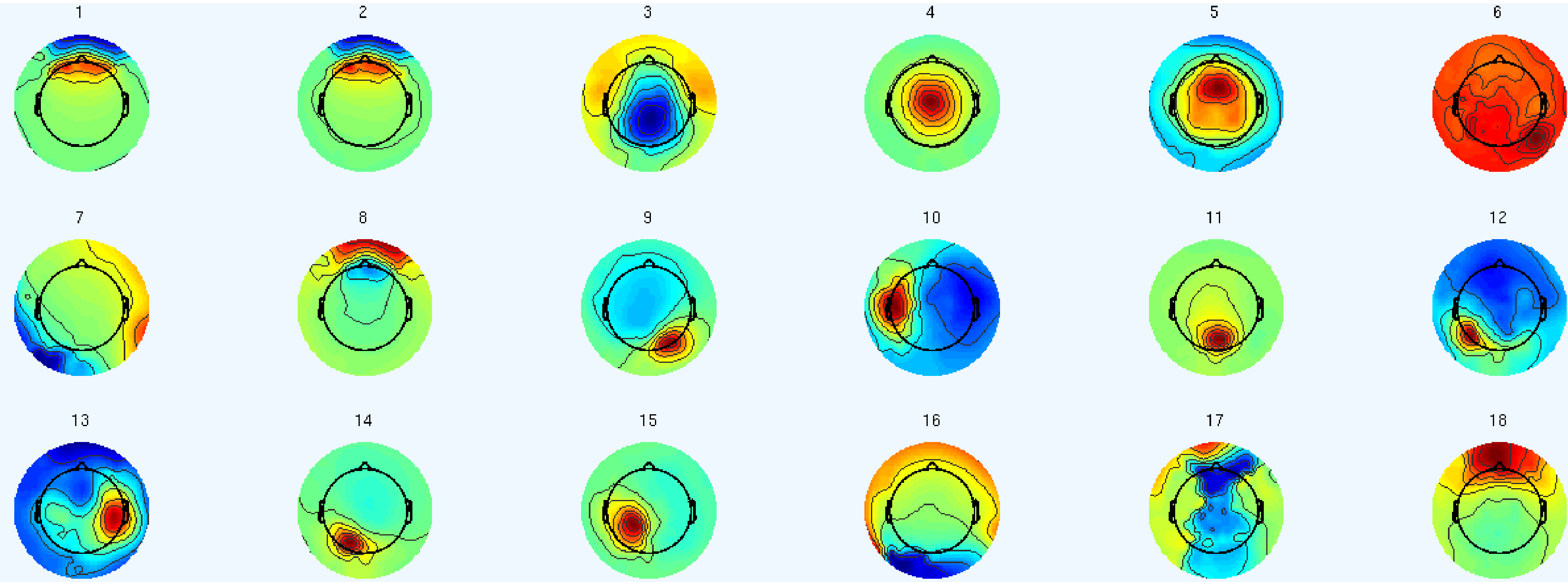
bt73 single model components



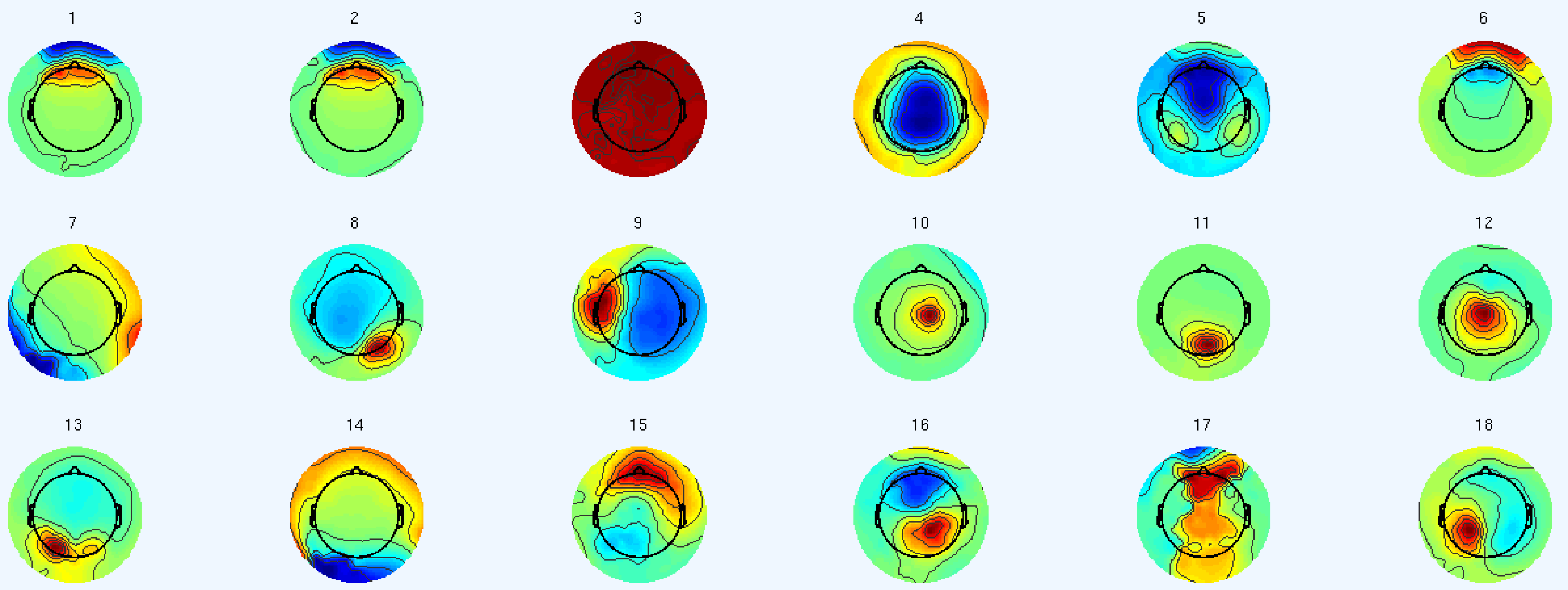
eb79 blue model components

- Prominent midline and occipital alpha components
- Weak mu components



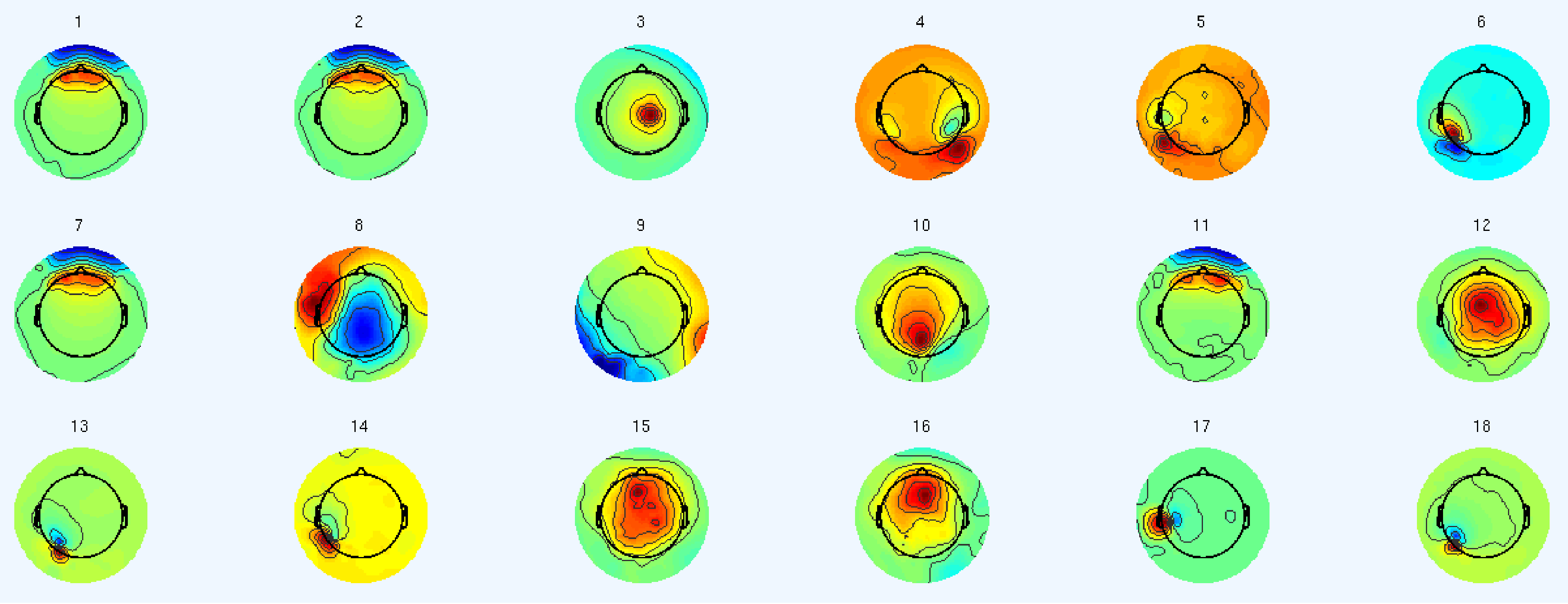
eb79 green model components

- Prominent occipital alpha components
- Weaker frontal midline

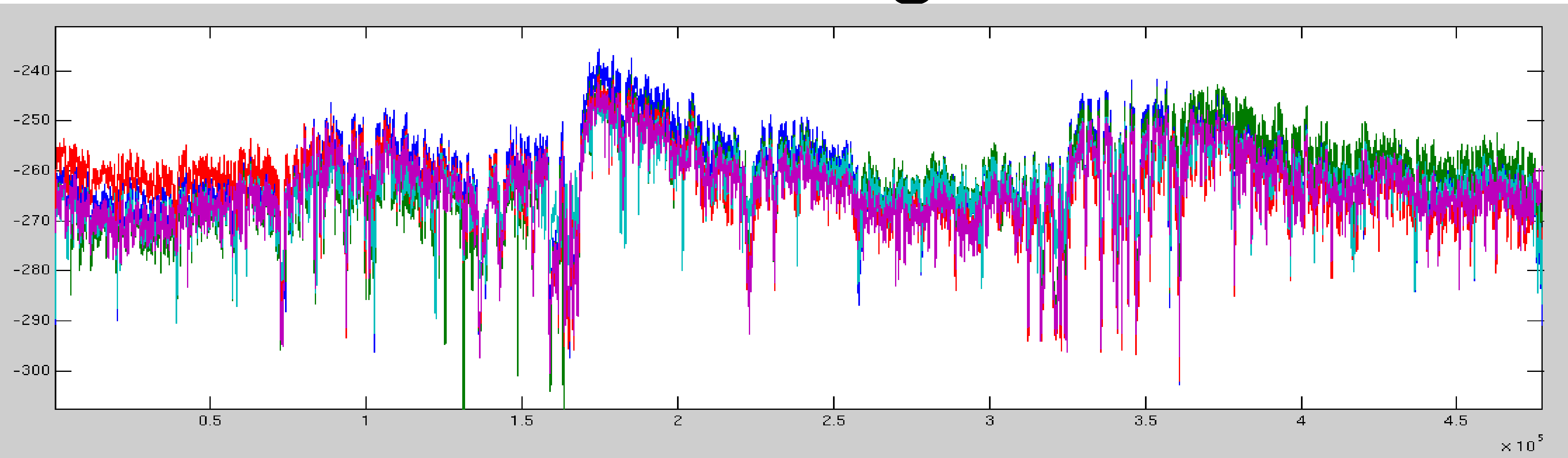


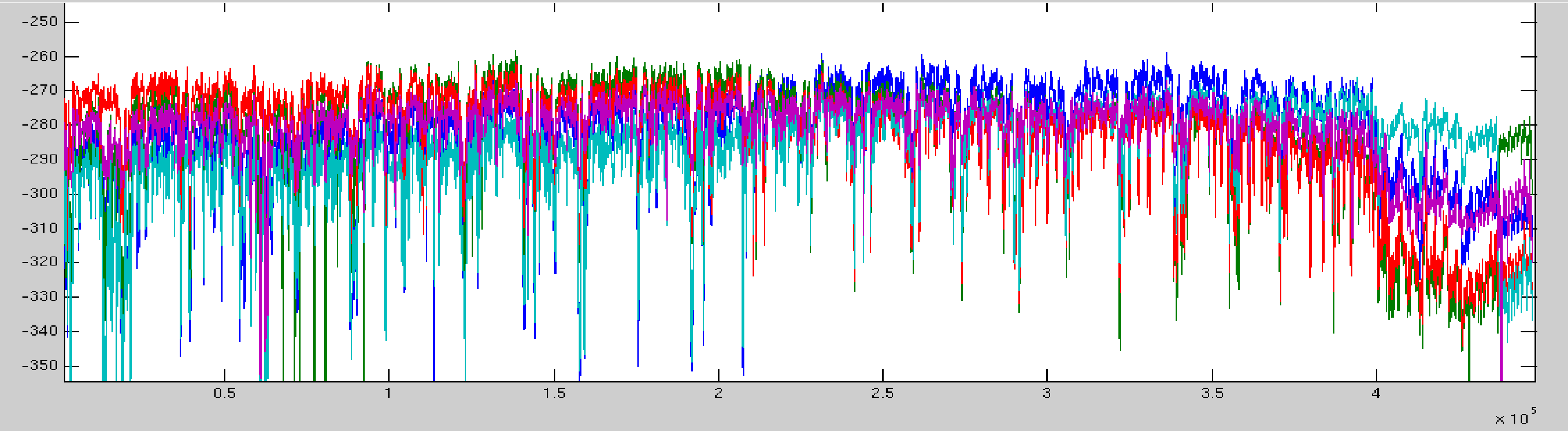
eb79 red model components

- Prominent muscle components (4, 5, 6, 13, 14, 17, 18)



ld81 and dh84 segmentation





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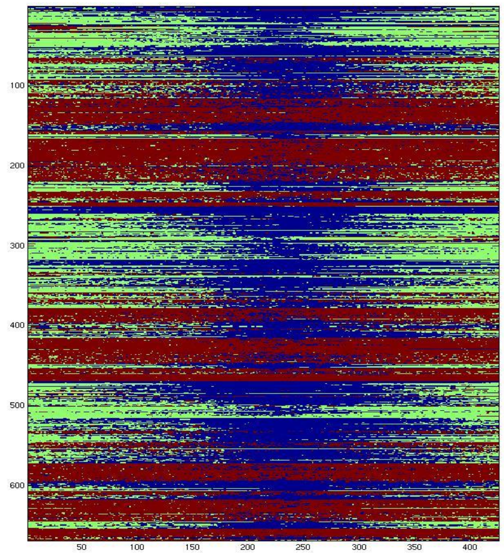
Consistency over Number of Models

3 models

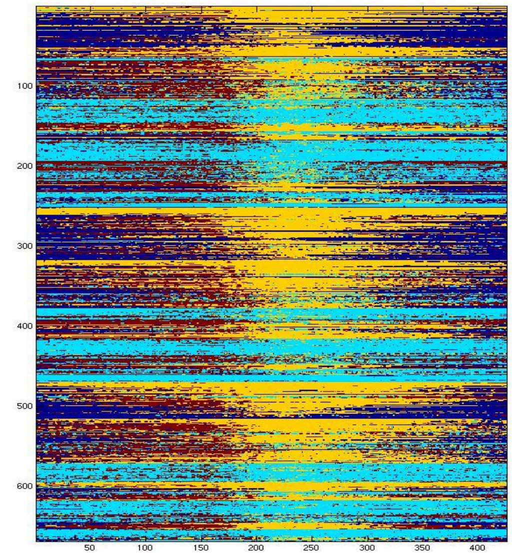
trial



time

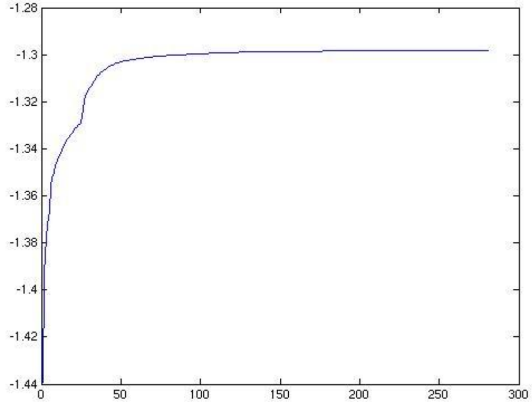
4 models

trial



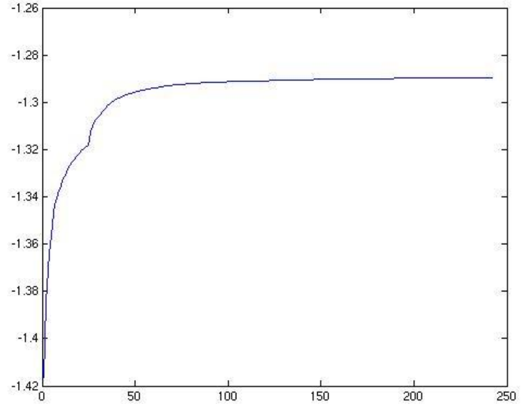
time

log likelihood



iteration

log likelihood



iteration

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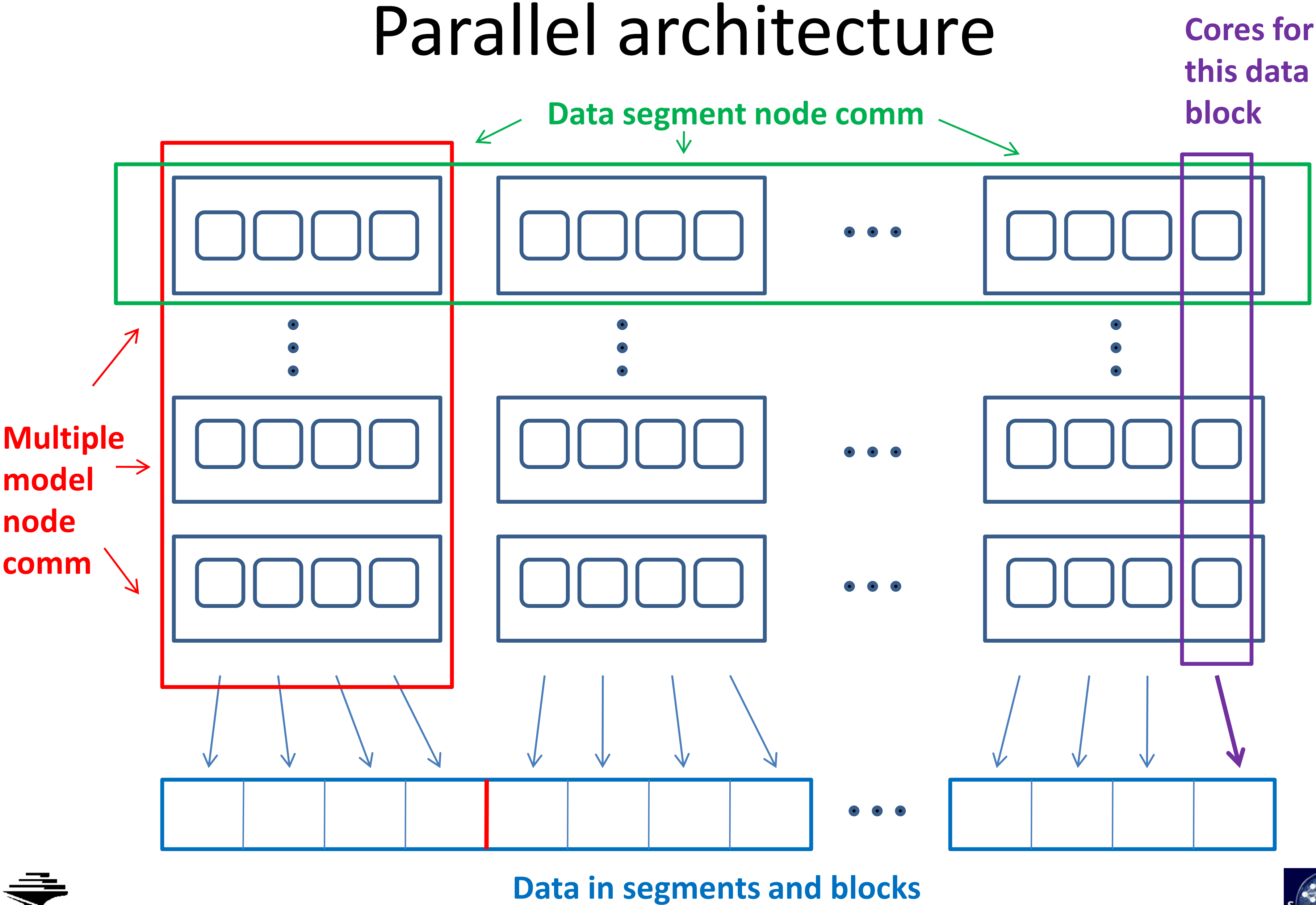
Parallel architecture



Parallel architecture (cont.)

- Parallelization is implemented using MPI to parallelize over nodes, and OpenMP to parallelize over cores within a node (using shared memory)
- Data is divided into segments (assigned to nodes) and blocks (assigned to cores)
- Multiple nodes are devoted to the same segment, one for each model
- An "update" is computed for each segment. Two directions of data communication flow:
  – Model nodes communicate to normalize update by likelihood of segment over all models
  – Segment nodes communicate to average the segment updates into one global update of parameters
- Global update computed at root node and sent back to model and segment nodes
- Also implemented with unstructured collection of cores for random assignment on large cluster
- Portable implementation allows execution on many platforms, including Teragrid, an NSF project with NCSA, SDSC, and others

Take Home Messages

- With sufficient amount of data, **multiple ICA models can be estimated simultaneously** and used overcome non-stationarity and segment data.
- **Newton method** significantly improves convergence rate, and conditioning in multiple model case.
- **Arbitrary source densities** modeled with non-Gaussian source mixture model.
- Likelihood can be conveniently used to **reject data**.
- **Some EEG sources really are stationary** (eyes, heartbeat, power line, frontal midline, mu, etc.) These should be identified across models to improve efficiency of estimation (in progress). Alpha components seem to be variable.

Code and Papers

- There is Matlab code available!
  - Generate toy mixture model data for testing
  - Full method implemented: mixture sources, mixture ICA, Newton
- Paper draft available, with derivation of mixture model Newton updates
- Download from:

http://sccn.ucsd.edu/~jason

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- Thanks for your attention!