Reference identification by BSS

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Introduction

EEG acquisition :

- electrical potentials on the scalp or inside the skull
- differentially measured to a non-nul time varying reference electrode (common reference)

Objective :

- estimate the zero-referenced potentials
- classical solutions: average montage, bipolar, laplacian
- recent approaches : BSS (Hu *et al.* , 2007, 2008)

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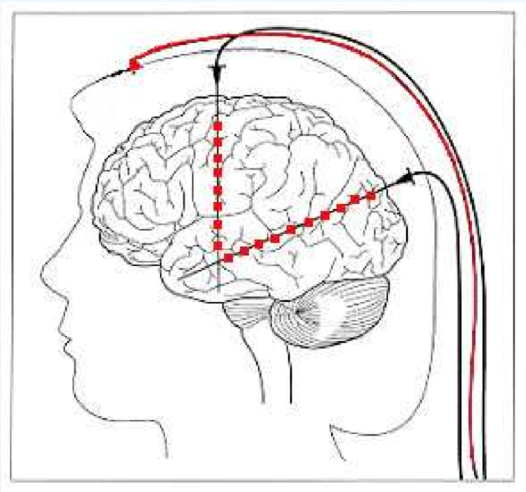
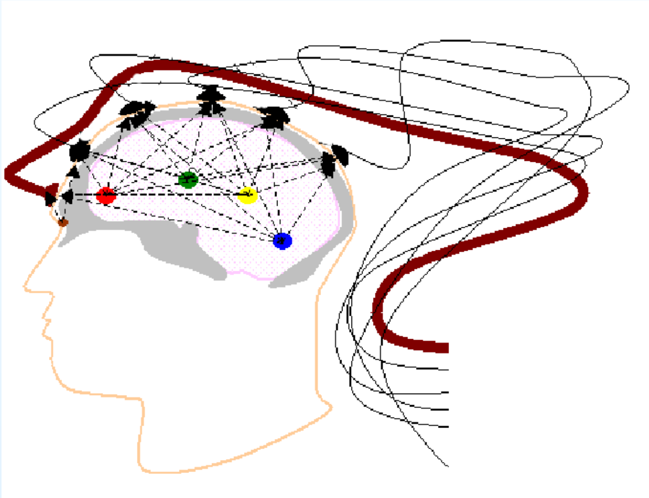


Independent reference



Mixed reference

Independent reference

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Classical BSS : linear mixing, (*zero referenced*)

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s} \tag{1}$$

- $\boldsymbol{x}$: M observed signals (EEG measured signals)
- $\boldsymbol{s}$: N 'independent' unknown sources
- $\boldsymbol{A}$: mixing matrix

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Independent common reference CR montage

$$\boldsymbol{x}_c = \boldsymbol{x} - r\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} & -1 \\ \boldsymbol{A} & \vdots \\ & -1 \end{bmatrix} \begin{bmatrix} \boldsymbol{s} \\ r \end{bmatrix} = \boldsymbol{Q}_c \begin{bmatrix} \boldsymbol{s} \\ r \end{bmatrix}, \qquad (2)$$

- $\boldsymbol{x}_c$ M measured signals, common reference (CR)
- $\boldsymbol{Q}_c$: constrained mixing matrix
- $\boldsymbol{s}, r$: $N+1$ 'independent' sources and reference (let $M = N+1$)

Estimating the reference: proposed algorithm

Ideal BSS solution: separation matrix $\boldsymbol{B}$ such as

$$\boldsymbol{B}\boldsymbol{Q}_c = \mathbb{I}_M$$

with

$$\boldsymbol{B} = \boldsymbol{U}\boldsymbol{W}$$

a matrix product between a rotation $\boldsymbol{U}$ and a whitening $\boldsymbol{W}$

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Basic idea:
$\Rightarrow$ estimate only the last row of $\boldsymbol{B}$, corresponding to r
$\Leftrightarrow$ estimate $\boldsymbol{W}$ *and* only the last row of $\boldsymbol{U}$: $\boldsymbol{u}_M$

1. Whitening $\boldsymbol{W}$

- eigen factorization of the full rank measurement covariance matrix of $\boldsymbol{x}_c$: $\boldsymbol{R}_c = \boldsymbol{V}\boldsymbol{\Sigma}\boldsymbol{V}^T$
- whitening matrix

$$\boldsymbol{W} = \boldsymbol{\Sigma}^{-1/2}\boldsymbol{V}^T \qquad (3)$$

Estimating the reference: proposed algorithm

2. Rotation $\boldsymbol{U}$

$$\boldsymbol{B}\boldsymbol{Q}_c = \mathbb{I}_M$$

$$\boldsymbol{U}\boldsymbol{W}\begin{bmatrix} & -1 \\ \boldsymbol{A} & \vdots \\ & -1 \end{bmatrix} = \mathbb{I}_M$$

$$\boldsymbol{U}\begin{bmatrix} & -\sum_{j=1}^{M} w(1,j) \\ \boldsymbol{W}\boldsymbol{A} & \vdots \\ & -\sum_{j=1}^{M} w(M,j) \end{bmatrix} = \mathbb{I}_M$$

$$\begin{bmatrix} & -\sum_{j=1}^{M} w(1,j) \\ \boldsymbol{W}\boldsymbol{A} & \vdots \\ & -\sum_{j=1}^{M} w(M,j) \end{bmatrix} = \boldsymbol{U}^T \quad (4)$$

Estimating the reference: proposed algorithm

Reference estimate

$$\boldsymbol{u}_M = [-1 \cdots -1]\boldsymbol{W}^T \tag{5}$$

$$\hat{r}^* = \boldsymbol{u}_M \boldsymbol{W} \boldsymbol{x}_c \tag{6}$$

$$\hat{r}^* = [-1 \cdots -1]\boldsymbol{R}_c^{-1} \boldsymbol{x}_c \tag{7}$$

Estimating the reference: proposed algorithm

Reference estimate

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Problem

- BSS implicit hypothesis $\rightarrow$ unit standard deviation for $\hat{r}^*$
- scaling: find the best gain α such as

$$\min_{\alpha} \mathbb{E}[(\alpha \hat{r}^* - r)^2]$$

Scaling

For any measured signal $x_{c,i}$:

$$\mathbb{E}[(\alpha_i \hat{r}^* - r)^2] = \mathbb{E}[(x_i - r + \alpha_i \hat{r}^*)^2] = \mathbb{E}[(x_{c,i} + \alpha_i \hat{r}^*)^2]$$
$$= \mathbb{E}[x_{c,i}^2] + \alpha_i^2 \mathbb{E}[\hat{r}^{*2}] + 2\alpha_i \mathbb{E}[x_{c,i}\hat{r}^*] \quad (8)$$
$$(9)$$

Minimize with respect to α_i:

$$\alpha_i = -\frac{\mathbb{E}[x_{c,i}\hat{r}^*]}{\mathbb{E}[\hat{r}^{*2}]} \quad (10)$$
$$= \left(-\frac{\mathbb{E}[(x_i - r)\hat{r}^*]}{\mathbb{E}[\hat{r}^{*2}]} = -\frac{\mathbb{E}[-r\hat{r}^*]}{\mathbb{E}[\hat{r}^{*2}]}\right)$$

independent of i

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independent of i

Scaling:

$$\alpha = -\frac{\mathbb{E}[x_{c,i}\hat{r}^*]}{\mathbb{E}[\hat{r}^{*2}]}, \qquad \hat{r} = \alpha \hat{r}^* \quad (11)$$

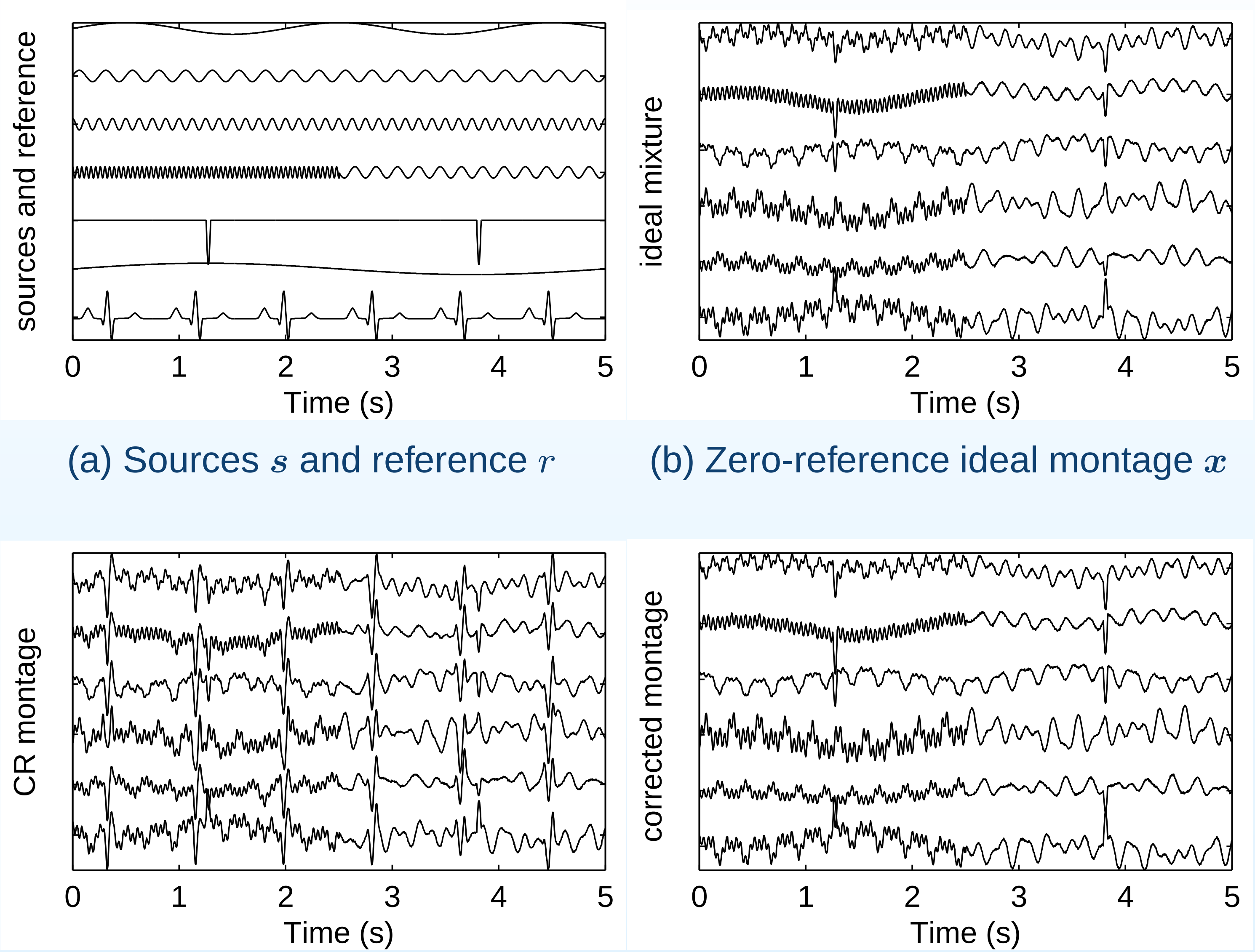
- Introduction

Independent reference
- Proposed algorithm
- Scaling
- Simulation
- Depth EEG application

Mixed reference

Thank you.

Simulation



(a) Sources s and reference r (b) Zero-reference ideal montage x

(c) CR montage x_c (d) Corrected montage $\hat{x}$

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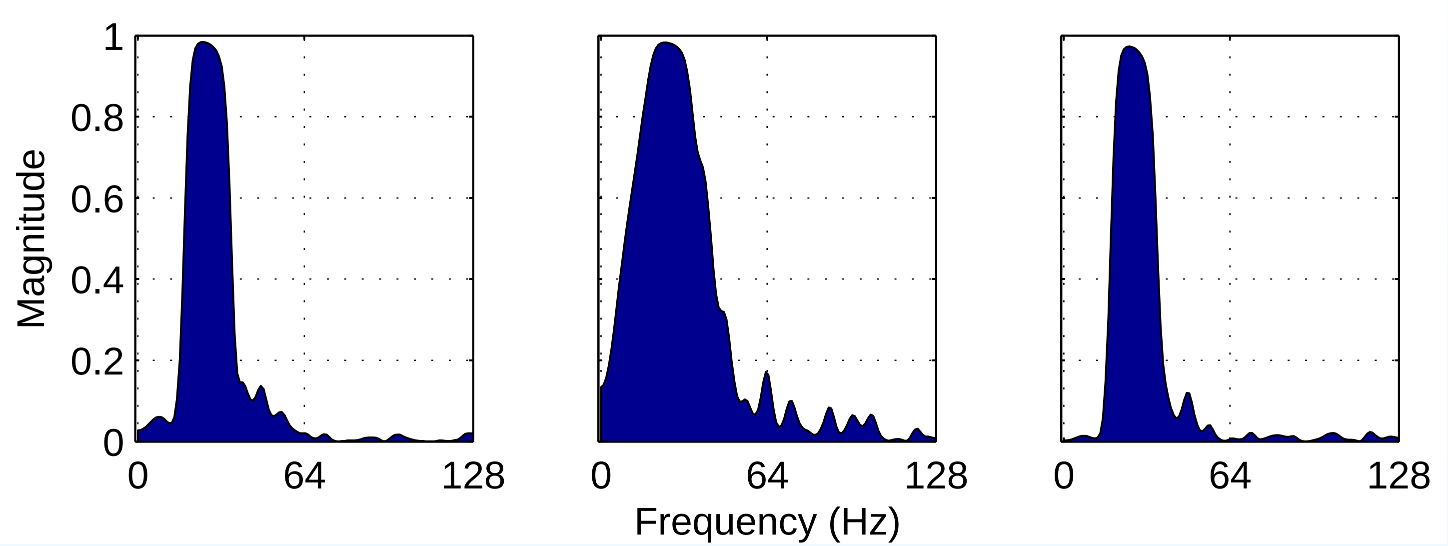
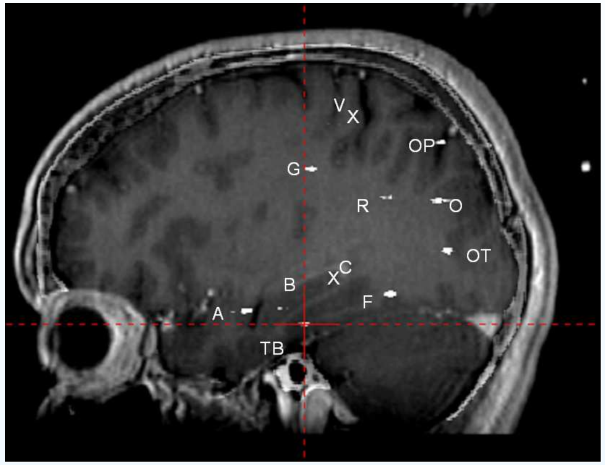
Thank you.

Possible application: coherence estimation

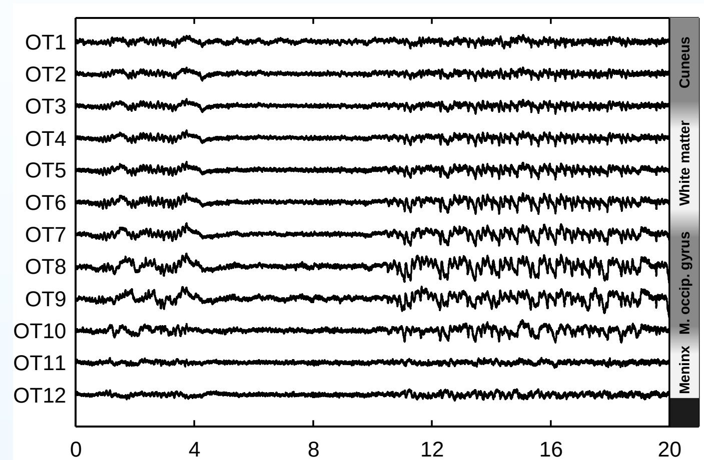


Coherence between signals 1 and 5 for the ideal zero-referenced mixture x (left), CR montage x_c (center) and corrected montage $\hat{x}$ (right).

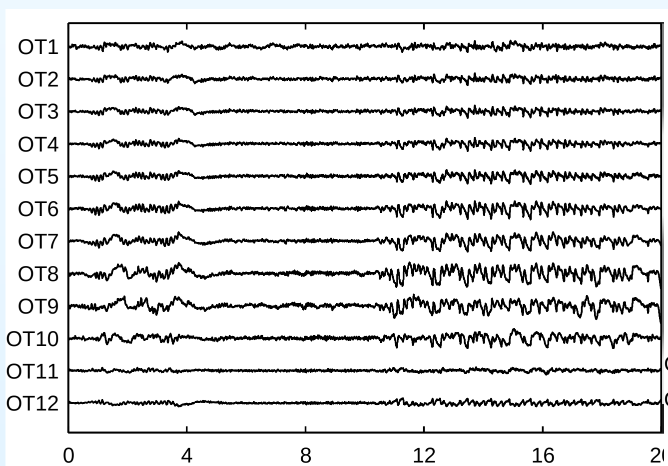
Depth EEG application



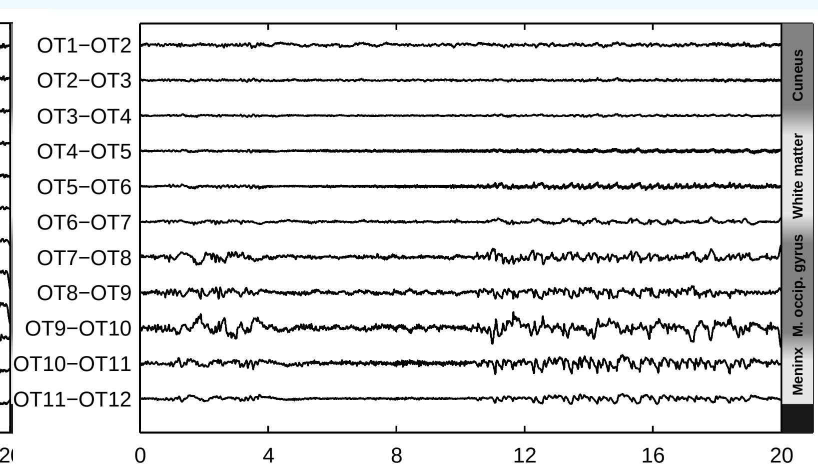
(e) Implantation



(f) original CR montage

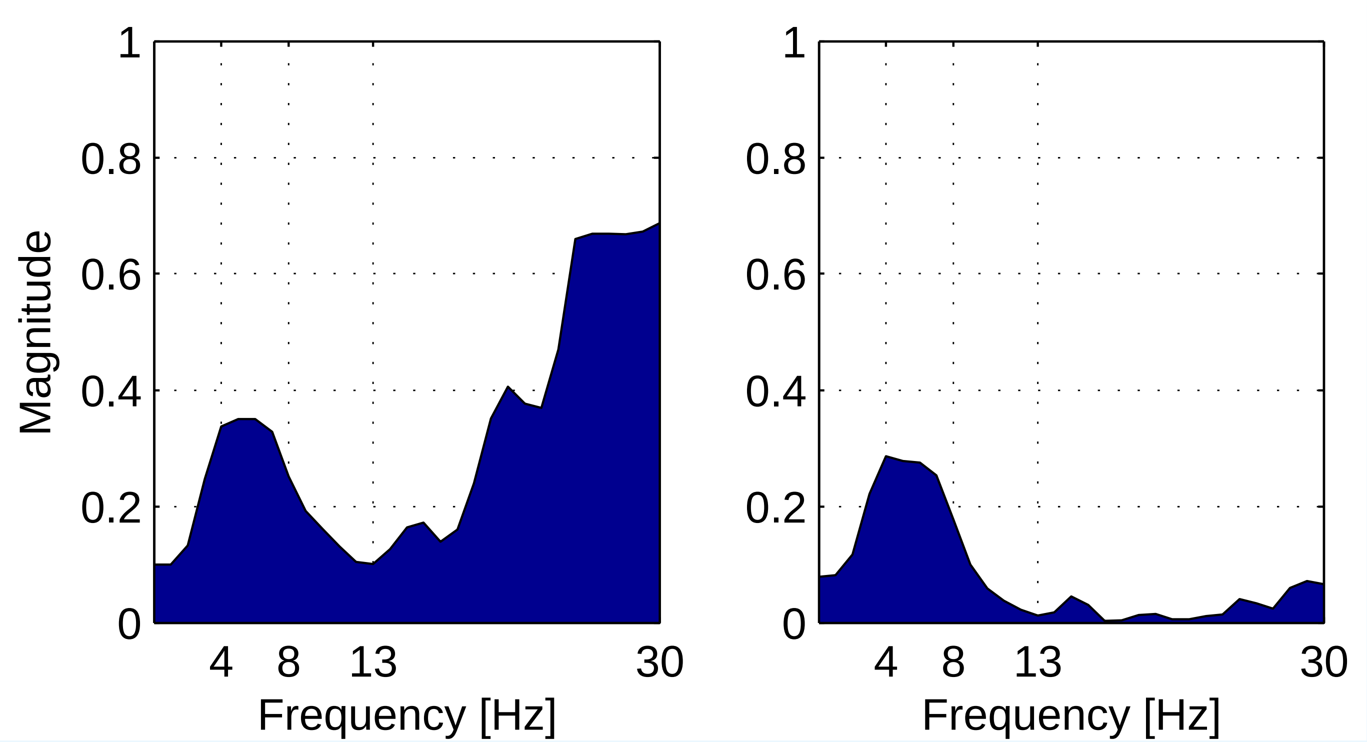


(g) corrected ZR montage



(h) bipolar BL montage

Coherence estimation



Coherence between channels OT_1 and OT_8, before and after correction

Mixed reference

Common reference montage

Noisy zero referenced potentials:

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n} \tag{12}$$

where $\boldsymbol{n}$ are independent gaussian noises

Head common reference realistic setup:

$$\boldsymbol{x}_{CRM} = \boldsymbol{T}_{CRM} \cdot \boldsymbol{x} \tag{13}$$

where $\boldsymbol{T}_{CRM}$ $(M - 1 \times M)$ is given by:

$$\boldsymbol{T}_{CRM} = \begin{bmatrix} 1 & 0 & \dots & 0 & -1 \\ 0 & 1 & \dots & 0 & -1 \\ \vdots & \vdots & \dots & \vdots & -1 \\ 0 & 0 & \dots & 1 & -1 \end{bmatrix}$$

Assume $N = M - 1$ sources (*i.e.*, enough measures).

Montages

All montages are obtained from the $\boldsymbol{x}_{CRM}$

- bipolar (augmented): $\boldsymbol{x}_{ABLM} = \boldsymbol{T}_{ABLM} \cdot \boldsymbol{x}_{CRM}$

$$\boldsymbol{T}_{ABLM} = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 & -1 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \quad (14)$$

- average (augmented): $\boldsymbol{x}_{AARM} = \boldsymbol{T}_{AARM} \cdot \boldsymbol{x}_{CRM}$

$$\boldsymbol{T}_{AARM} = \begin{bmatrix} 1-\frac{1}{M-1} & -\frac{1}{M-1} & \dots & -\frac{1}{M-1} \\ -\frac{1}{M-1} & 1-\frac{1}{M-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\frac{1}{M-1} \\ -\frac{1}{M-1} & \dots & -\frac{1}{M-1} & 1-\frac{1}{M-1} \\ -\frac{1}{M-1} & -\frac{1}{M-1} & -\frac{1}{M-1} & -\frac{1}{M-1} \end{bmatrix} \quad (15)$$

Problems

- are the source separation solutions equivalent ?

  No. The noise affecting the montages is not equivalent ! AARM gives the best solution (separability index, correlation with the original sources, correlation with the ECG in real recordings).

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- which is the best approximation of the zero referenced montage?

$$\boldsymbol{T}_{AARM} = \boldsymbol{T}^{+}_{CRM}$$

  The AARM (augmented average reference montage) is the closest solution (MSE) to the reference problem !

Conclusion and future research

Independent reference

- validate the method on several depth EEG recordings
- extra-cranial reference for surface EEG ?
- connectivity estimation
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Mixed reference

- use the augmented average montage

References:

- independent reference : IEEE EMBC'10 (Buenos Aires)
- mixed reference : Biomedical Signal Processing & Control (Elsevier) - minor revision

`http://perso.ensem.inpl-nancy.fr/Radu.Ranta/publications.html`

Thank you.